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ERRATUM: ASYMPTOTIC PROFILES OF BASIC REPRODUCTION NUMBER FOR EPIDEMIC SPREADING IN HETEROGENEOUS ENVIRONMENT*

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Abstract. The main purpose of this erratum is to correct some errors of signs in [S. Chen and J. Shi, *SIAM J. Appl. Math.*, 80 (2020), pp. 1247–1271].

Key words. basic reproduction number, reaction-diffusion, heterogeneous environment, compartmental epidemic models

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There is an extra negative sign in the statement and some parts of the proof of Theorem 3.1 of the original publication of [1]. The correct statement of [1, Theorem 3.1] should be as follows.

THEOREM 3.1. *Assume that* (A1)–(A5) *and* (A7) *hold,*

$$s\left(-\overline{V}_0^c\right) < 0,\ s\left(-\underline{V}_0^c\right) < 0 \ \text{ and } \ r\left(\left(\overline{V}_0^c\right)^{-1}\underline{F}_0^c\right) > 0, \tag{3.4}$$

and there exists $\epsilon_0 > 0$ *such that* $-\overline{V}_{\epsilon_0}^c$ *is cooperative and* $\underline{F}_{\epsilon_0}^c$ *is positive, where* $\underline{V}_\epsilon^c$, $\overline{V}_\epsilon^c$, *and* $\underline{F}_\epsilon^c$ *are defined in* (3.3). *If the matrix* $-V(x, c(x)) + aF(x, c(x))$ *is irreducible for any* $a > 0$ *and* $x \in \overline{\Omega}$, *where* $c(x)$ *is defined in* (3.2), *then*

$$\lim_{(d_1,\dots,d_n)\to(0,\dots,0)} R_0 = R_0^c := \max_{x\in\overline{\Omega}}\left[r\left(V^{-1}(x, c(x))F(x, c(x))\right)\right]. \tag{3.5}$$

In the proof of Theorem 3.1 some negative signs should be removed or added as follows.

1. Page 1258 (lines 9, 15, and 17):

$$r\left(-V^{-1}(x, c(x))F(x, c(x))\right) \ \text{ should be } \ r\left(V^{-1}(x, c(x))F(x, c(x))\right).$$

2. Equation (3.9), page 1256 (line -9), equation (3.13), and page 1258 (line -12):

$$\left(d_I\Delta - \underline{V}_\epsilon^x\right)^{-1}\overline{F}_\epsilon^x \ \text{ should be } \ -\left(d_I\Delta - \underline{V}_\epsilon^x\right)^{-1}\overline{F}_\epsilon^x.$$

3. Equation (3.9), and page 1258 (line -12):

$$\left(d_I\Delta - \overline{V}_\epsilon^x\right)^{-1}\underline{F}_\epsilon^x \ \text{ should be } \ -\left(d_I\Delta - \overline{V}_\epsilon^x\right)^{-1}\underline{F}_\epsilon^x.$$

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The result of Theorem 3.1 also affects the one in Proposition 3.3. The correct statement of [1, Proposiotion 3.3] should be as follows.

Proposition 3.3. *Assume that* (A1)–(A5), (A7)–(A8), *and* (3.17) *hold, and there exists* $\epsilon_0 > 0$ *such that* $-\overline{V}_{\epsilon_0}$ *is cooperative and* $\underline{F}_{\epsilon_0}$ *is positive, where* $\underline{V}_\epsilon$, $\overline{V}_\epsilon$, *and* $\underline{F}_\epsilon$ *are defined in* (3.16). *If the matrix* $-V(x,\tilde{u}) + aF(x,\tilde{u})$ *is irreducible for any* $a > 0$ *and* $x \in \overline{\Omega}$, *where* $\tilde{u}$ *is defined in* (3.15), *then*

$$\lim_{(d_1,\dots,d_m,d_{m+1},\dots,d_n)\to(0,\dots,0,\infty,\dots,\infty)} R_0 = R_0^{\tilde{u}} := \max_{x\in\overline{\Omega}} \left[r\left(V^{-1}(x,\tilde{u})F(x,\tilde{u})\right)\right],$$

where $(d_1,\dots,d_m,d_{m+1},\dots,d_n) \to (0,\dots,0,\infty,\dots,\infty)$ *means* $\max_{1\le j\le m} d_j \to 0$ *and* $\min_{m+1\le j\le n} d_j \to \infty$.

The changes in the statements in Theorem 3.1 and Proposition 3.3 do not affect other results of the paper, including the applications of Theorems 3.1 and 3.2 and Propositions 3.3 and 3.4 in section 4.

REFERENCE

[1] S. Chen and J. Shi, *Asymptotic profiles of basic reproduction number for epidemic spreading in heterogeneous enviroment*, SIAM J. Appl. Math., 80 (2020), pp. 1247–1271.