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NONEXISTENCE OF NONCONSTANT POSITIVE STEADY STATES OF A DIFFUSIVE PREDATOR-PREY MODEL WITH FEAR EFFECT

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Abstract In this paper, we investigate a diffusive predator-prey model with fear effect. It is shown that, for the linear predator functional response case, the positive constant steady state is globally asymptotically stable if it exists. On the other hand, for the Holling type II predator functional response case, it is proved that there exist no nonconstant positive steady states for large conversion rate. Our results limit the parameters range where complex spatiotemporal pattern formation can occur.

Keywords Reaction-diffusion, fear effect, flobal stability, nonexistence of nonconstant steady states.

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1. Introduction

The interaction between predator and prey is one of fundamental ecological phenomena. Adding the random movement in the spatial habitat, reaction-diffusion systems have been used to described the interaction and dispersal of the predator and prey species [1,6,7,15,17,18,21,22]. Recently some researchers found that the fear of the predators could lead to the reduction of the prey, see [8–10,19,23] and references therein. A reaction-diffusion predator-prey system with fear effect and predator-taxis is proposed in [20]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1 \Delta u + \alpha \nabla \cdot (\beta(u) u \nabla v) + \dfrac{ru}{1+kv} - du - au^2 - \dfrac{buv}{1+qu}, & x \in \Omega,\ t>0, \\ \dfrac{\partial v}{\partial t} = d_2 \Delta v - m_1 v - m_2 v^2 + \dfrac{cuv}{1+qu}, & x \in \Omega,\ t>0, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega,\ t>0, \\ u(x,0) = u_0(x) \geq (\not\equiv) 0,\ v(x,0) = v_0(x) \geq (\not\equiv) 0, & \end{cases} \tag{1.1}$$

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where $u(x,t)$ and $v(x,t)$ are the density functions of the prey and predator population; Ω is a bounded domain in $\mathbb{R}^N (N \leq 3)$ with a smooth boundary $\partial\Omega$; d_1 and d_2 are the diffusion coefficients of the prey and predator respectively, and $\alpha\nabla \cdot (\beta(u)u\nabla v)$ represent the predator-taxis that predator moves toward high prey concentration location; $m_1 > 0$ and $m_2 \geq 0$ account for the death rate and crowding effect of the predator, $r > 0$ and $d > 0$ are the birth and death rates of the prey respectively, and $a > 0$ reflects the intro-species competition of the prey; $b > 0$ and $c > 0$ measure the interaction strength between the predator and prey; $q \geq 0$ measures the prey's ability to evade attack and $u/(1+qu)$ is the Holling type II functional response; and $k > 0$ represents the fear effect. For the corresponding kinetic model, it is known that high levels of fear can stabilize the positive steady state, and low levels of fear can induce multiple limit cycles leading to bistable phenomenon [19]. For the diffusive model (1.1) with $q = 0$, it is shown that the unique positive constant steady state is globally asymptotically stable under certain conditions, and for $q \neq 0$, complex spatiotemporal pattern formation can occur [20].

In this paper, we revisit model (1.1) without considering the predator-taxis, that is,

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1\Delta u + \dfrac{ru}{1+kv} - du - au^2 - \dfrac{buv}{1+qu}, & x \in \Omega,\ t > 0, \\ \dfrac{\partial v}{\partial t} = d_2\Delta v - m_1 v - m_2 v^2 + \dfrac{cuv}{1+qu}, & x \in \Omega,\ t > 0, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x) \geq (\not\equiv) 0,\ v(x,0) = v_0(x) \geq (\not\equiv) 0. \end{cases} \tag{1.2}$$

We find that, for $q = 0$ (Lotka-Volterra case), the positive constant steady state is globally asymptotically stable if exists, and for $q \neq 0$ (Holling Type II case), there exists no nonconstant positive steady states with large conversion rate c. Our result for global stability is proved under weaker condition that the ones in [20] but also without predator-taxis. Our results give some ranges for the model parameters within which, spatiotemporal pattern formation cannot occur, and supplement some results obtained in [20].

The model (1.2) is a variant of more commonly studied Rosenzweig-MacArthur predator-prey model with Holling type II functional response [13, 18, 22]. By using conversion rate c as a variable parameter, they showed the existence of Hopf and steady state bifurcations, and there exist no nonconstant steady states when c is large or small, which implies that the global bifurcating branches of steady state solutions of system are bounded loops. Related results were also obtained for the diffusive predator-prey model with Holling type III predator functional response [2, 16], or other more general functional responses [4], or other growth functions [5], or delay effect [3].

The rest of the paper is organized as follows. In Section 2, we consider the global stability of the constant positive steady state for the Lotka-Volterra predation case. In Section 3, we show the nonexistence of nonconstant positive steady states for the Holling type II predation case.

2. The Lotka-Volterra case

In this section, we show that, when $q = 0$, the constant positive steady state of model (1.2) is globally asymptotically stable if it exists. Therefore, complex pattern formation cannot occur. Clearly, for $q = 0$, model (1.2) has a constant positive steady state (u_*, v_*) if and only if $r > d + \frac{am_1}{c}$, see [20]. Then we have:

Theorem 2.1. *Assume that $r > d + \dfrac{am_1}{c}$ and $q = 0$. Then system (1.2) has a unique positive constant (u_*, v_*) which is globally asymptotically stable.*

Proof. Let $h(v) = \dfrac{krv}{1+kv} + bv$, and construct a Lyapunov functional as follows:

$$V(u,v) = c\int_\Omega \int_{u_*}^{u} \frac{\xi - u_*}{\xi} d\xi dx + \int_\Omega \int_{v_*}^{v} \frac{h(\eta) - h(v_*)}{\eta} d\eta dx. \tag{2.1}$$

If $(u(x,t), v(x,t))$ is a solution of system (1.2), then

$$\begin{aligned}\frac{dV(u(x,t),v(x,t))}{dt} =& c\int_\Omega (u-u_*)[au_* + h(v_*) - au - h(v)]dx \\ &+ \int_\Omega [h(v) - h(v_*)](-cu_* + m_2 v_* - m_2 v + cu)dx \\ &- cd_1 u_* \int_\Omega \frac{|\nabla u|^2}{u^2} dx - d_2 \int_\Omega \frac{h'(v)v - h(v) + h(v_*)}{v^2} |\nabla v|^2 dx \\ =& - cd_1 u_* \int_\Omega \frac{|\nabla u|^2}{u^2} dx - d_2 \int_\Omega \frac{h'(v)v - h(v) + h(v_*)}{v^2} |\nabla v|^2 dx \\ &- ca \int_\Omega (u-u_*)^2 dx - m_2 \int_\Omega (v - v_*)[h(v) - h(v_*)]dx.\end{aligned} \tag{2.2}$$

Since $h'(v) > 0$ and $h''(v) < 0$ for $v \in (0, \infty)$, it follows that

$$\frac{dV(u(x,t),v(x,t))}{dt} \leq 0,$$

and the equality holds if and only if $u(x,t) = u_*$ and $v(x,t) = v_*$. Therefore, (u_*, v_*) is globally asymptotically stable from LaSalle Invariance Principle. $\square$

In [20] Theorem 4.2, a global stability result was proved under extra condition for the predator-taxis case. Here we prove the global stability of constant steady state holds whenever it exists.

For the sake of completeness, we also describe the dynamics of (1.2) for $r < d + \frac{am_1}{c}$ and $q = 0$ in the following. The main method is the comparison principle, and here we omit the proof.

Theorem 2.2. *Assume that $q = 0$. Then*

1. *if $d < r < d + \dfrac{am_1}{c}$, then the prey-only constant steady state $(\dfrac{r-d}{a}, 0)$ is globally asymptotically stable;*
2. *if $0 < r < d$, then the trivial steady state $(0,0)$ is globally asymptotically stable.*

From Theorems 2.1 and 2.2, the dynamics of (1.2) is completely classified when $q = 0$, which is similar to the classical Lotka-Volterra predator-prey model.

3. The Holling type II case

In this section, we consider the case of Holling type II functional response ($q \neq 0$), and we investigate the positive steady states of (1.2) for large c, which satisfy the following system

$$\begin{cases} -d_1\Delta u = \dfrac{ru}{1+kv} - du - au^2 - \dfrac{buv}{1+qu}, & x \in \Omega, \\ -d_2\Delta v = -m_1 v - m_2 v^2 + \dfrac{cuv}{1+qu}, & x \in \Omega, \\ \partial_n u = \partial_n v = 0, & x \in \partial\Omega. \end{cases} \tag{3.1}$$

It follow from the first equation of model (3.1) that

$$(r-d)\int_\Omega u dx > \int_\Omega (\frac{ru}{1+kv} - du) dx > 0,$$

which implies that $r > d$ if $u(x)$ and $v(x)$ are positive. Therefore, if $r < d$, then system (3.1) has no positive solutions, and we will only consider the case of $r > d$ in the following.

By virtue of the transformation $w = cu$, $z = bv$ and $\rho = 1/c$, we see that (w, z) satisfies

$$\begin{cases} -d_1\Delta w = \dfrac{rbw}{b+kz} - dw - a\rho w^2 - \dfrac{wz}{1+q\rho w}, & x \in \Omega, \\ -d_2\Delta z = -m_1 z - \dfrac{m_2}{b} z^2 + \dfrac{wz}{1+q\rho w}, & x \in \Omega, \\ \partial_n w = \partial_n z = 0, & x \in \partial\Omega. \end{cases} \tag{3.2}$$

Then the nonexistence of positive solutions of system (3.1) for large c is equivalent to that of system (3.2) for small ρ. We first sketch the main steps to prove the nonexistence, and the method is motivated by the one in [13]:

Step 1: We show that, for $\rho = 0$, system (3.2) has a unique positive solution, which is constant and non-degenerate;

Step 2: We show that, if $(w_i(x), z_i(x))$ is a positive solution of system (3.2) for $\rho = \rho_i$, where $i = 1, 2, \cdots$, and $\lim_{i\to\infty} \rho_i = 0$, then there exists a subsequence $\{i_k\}_{k=1}^\infty$ such that $(w_{i_k}(x), z_{i_k}(x)) \to (\tilde{w}(x), \tilde{z}(x))$ in $C^2(\overline{\Omega})$ as $k \to \infty$, where $(\tilde{w}(x), \tilde{z}(x))$ is a positive solution of system (3.2) for $\rho = 0$.

Then it follow from the implicit function theorem that system (3.2) has no nonconstant positive solutions for small ρ.

We first prove Step 1.

Proposition 3.1. *Assume that $r > d$ and $\rho = 0$. Then system* (3.2) *has a unique positive solution (w_*, z_*), where z_* is the unique positive root of*

$$\frac{rb}{b+kz} - d - z = 0,$$

and $w_* = m_1 + \dfrac{m_2}{b} z_*$.

Proof. It is easy to verify that (w_*, z_*) is the unique constant positive solution of system (3.2) for $\rho = 0$. Let

$$V_1(w,z) = \int_\Omega \left\{ \frac{w-w_*}{w} \left[d_1 \Delta w + w \left(\frac{rb}{b+kz} - d - z \right) \right] \right\} dx + \int_\Omega \left\{ \frac{h_1(z)-h_1(z_*)}{z} \left[d_2 \Delta z + z \left(-m_1 - \frac{m_2}{b} z + w \right) \right] \right\} dx,$$

where

$$h_1(z) = z + \frac{krz}{b+kz}.$$

As in the proof of Theorem 2.1, we calculate that

$$V_1(w,z) = - d_1 w_* \int_\Omega \frac{|\nabla w|^2}{w^2} dx - d_2 \int_\Omega \frac{h_1'(z)z - h_1(z) + h_1(z_*)}{z^2} |\nabla z|^2 dx - \frac{m_2}{b} \int_\Omega (z-z_*)[h_1(z)-h_1(z_*)] dx.$$

Noticing that $h_1'(v) > 0$ and $h_1''(v) < 0$ for $v \in (0,\infty)$, we have $V_1(w,z) \le 0$. If $(w(x), z(x))$ is a positive solution of system (3.2) for $\rho = 0$, then $V_1(w,z) = 0$, which implies that $w(x)$ and $z(x)$ are constant. This completes the proof. $\square$

For Step 2, we need to use the following two well-known results. The first one is from [11].

Lemma 3.1 (Lemma 2.1, [11])**.** *Assume that Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary, and d is a nonnegative constant. If $z \in W^{1,2}(\Omega)$ is a non-negative weak solution of the following inequalities*

$$\begin{cases} -\Delta z + dz \ge 0, & x \in \Omega, \\ \partial_\nu z \le 0, & x \in \partial\Omega, \end{cases}$$

then, for any $q \in \left[1, \frac{N}{N-2}\right)$, there exists a positive constant C such that

$$\|z\|_q \le C \inf_{x\in\Omega} z,$$

where C is determined only by q, d and Ω.

The second one is a Harnack inequality from [12, 14].

Lemma 3.2 (Lemma 2.2, [12])**.** *Assume that Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary, and $c(x) \in L^q(\Omega)$ for some $q > N/2$. If $z \in W^{1,2}(\Omega)$ is a non-negative weak solution of the following problem*

$$\begin{cases} \Delta z + c(x)z = 0, & x \in \Omega, \\ \partial_\nu z = 0, & x \in \partial\Omega, \end{cases}$$

then, there exists a positive constant C such that

$$\sup_{x\in\Omega} z \le C \inf_{x\in\Omega} z,$$

where C is determined only by $\|c(x)\|_q$, q, and Ω.

Based on Lemmas 3.1 and 3.2, we have the following a *priori* estimate for the positive solutions of system (3.2).

Proposition 3.2. *Assume that $r > d$, and $(w_i(x), z_i(x))$ is a positive solution of system (3.2) for $\rho = \rho_i$, where $i = 1, 2, \cdots$, and $\lim_{i\to\infty} \rho_i = 0$. Then there exists a subsequence $\{i_k\}_{k=1}^{\infty}$ such that $(w_{i_k}(x), z_{i_k}(x)) \to (\tilde{w}(x), \tilde{z}(x))$ in $C^2(\overline{\Omega})$ as $k \to \infty$, where $(\tilde{w}(x), \tilde{z}(x))$ is a positive solution of system (3.2) for $\rho = 0$.*

Proof. We first show the existence of the upper bounds for $\{w_i(x)\}_{i=1}^{\infty}$ and $\{z_i(x)\}_{i=1}^{\infty}$. For $m_2 = 0$, we have

$$-d_2\Delta z_i + m_1 z_i = \frac{w_i z_i}{1 + q\rho_i w_i} > 0,$$

and it follows from Lemma 3.1 that there exists a positive constant C_1 such that

$$\|z_i\|_2 \le C_2 \inf_{\Omega} z_i \quad \text{for all} \quad i \ge 1. \tag{3.3}$$

We claim that there exists a positive constant C_2 such that

$$\|z_i\|_2 \le C_2 \quad \text{for all} \quad i \ge 1. \tag{3.4}$$

If it is not true, then there exists a subsequence $\{i_k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} \|z_{i_k}\|_2 = \infty$, which implies that $z_{i_k} \to \infty$ uniformly on $\overline{\Omega}$ as $k \to \infty$ from Eq. (3.3). Note that $0 < \rho_i w_{i_k} \le \frac{r-d}{a}$ from the comparison principle. Then, for sufficiently large k,

$$\int_{\Omega} w_{i_k} \left(\frac{rb}{b + kz_{i_k}} - d - a\rho_{i_k} w_{i_k} - \frac{z_{i_k}}{1 + q\rho_{i_k} w_{i_k}} \right) dx < 0,$$

which is a contradiction. Therefore, Eq. (3.4) holds.

For $m_2 \ne 0$, it follows from system (3.2) that

$$\begin{aligned} &m_1 \int_{\Omega} z_i dx + \frac{m_2}{b} \int_{\Omega} z_i^2 dx = \int_{\Omega} \frac{w_i z_i}{1 + q\rho_i w_i} dx \le \int_{\Omega} \frac{rbw_i}{b + kz_i} dx, \\ &\int_{\Omega} \frac{w_i}{1 + q\rho_i w_i} dx - \int_{\Omega} m_1 dx - \frac{m_2}{b} \int_{\Omega} z_i dx = -d_2 \int_{\Omega} \frac{1}{z_i^2} |\nabla z_i|^2 dx \le 0. \end{aligned} \tag{3.5}$$

This, combined with the fact that $0 < \rho_i w_i \le \frac{r-d}{a}$, implies that

$$\frac{m_2}{b} \|z_i\|_2^2 \le rbm_1 |\Omega| \left[1 + \frac{q(r-d)}{a} \right] + rm_2 |\Omega|^{\frac{1}{2}} \left[1 + \frac{q(r-d)}{a} \right] \|z_i\|_2.$$

Therefore, Eq. (3.4) also holds.

Then there exists a positive constant C_3 such that

$$\left\| \frac{rb}{b + kz_i} - d - a\rho_i w_i - \frac{z_i}{1 + q\rho_i w_i} \right\|_2 \le C_3 \quad \text{for all} \quad i \ge 1,$$

and from Lemma 3.2, we see that there exists a positive constant C_4 such that

$$\sup_{\Omega} w_i \le C_4 \inf_{\Omega} w_i \quad \text{for all} \quad i \ge 1. \tag{3.6}$$

We claim that there exists a positive constant C_5 such that

$$\sup_{\Omega} w_i \le C_5 \quad \text{for all} \quad i \ge 1. \tag{3.7}$$

By way of contradiction, there exists a subsequence $\{i_k\}_{k=1}^{\infty}$ such that

$$\lim_{k\to\infty} \sup_{\Omega} w_{i_k} = \infty,$$

which implies that $w_{i_k} \to \infty$ uniformly on $\overline{\Omega}$ as $k \to \infty$ from Eq. (3.6). It follows from Eq. (3.4) and the second equation of (3.5) that

$$\frac{a}{a+q(r-d)} \inf_{\Omega} w_i \le m_1 + \frac{m_2}{b}|\Omega|^{-\frac{1}{2}} C_2,$$

which is a contraction, and consequently Eq. (3.7) holds. Similarly, by virtue of Lemma 3.2 and Eq. (3.7), we see that there exists a positive constant C_6 such that

$$\sup_{\Omega} z_i \le C_6 \inf_{\Omega} z_i \quad \text{for all} \quad i \ge 1. \tag{3.8}$$

Then, it follows from the first equation of (3.5) that

$$m_1 \inf_{\Omega} z_i \le rb \sup_{\Omega} w_i \le rbC_5 \quad \text{for all} \quad i \ge 1.$$

This, combined with Eq. (3.8), implies that there exists a positive constant C_7 such that

$$\sup_{\Omega} z_i \le C_7 \quad \text{for all} \quad i \ge 1. \tag{3.9}$$

Now, we derive the lower bounds for $\{w_i(x)\}_{i=1}^{\infty}$ and $\{z_i(x)\}_{i=1}^{\infty}$. We claim that there exists a positive constant C_8 such that

$$\inf_{\Omega} w_i \ge C_8 \quad \text{for all} \quad i \ge 1. \tag{3.10}$$

If it is not true, then there exists a subsequence $\{i_k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} \inf_{\Omega} w_{i_k} = 0$. This, combined with Eq. (3.6), implies that $w_{i_k} \to 0$ uniformly on $\overline{\Omega}$ as $k \to \infty$. Then, for sufficiently large k,

$$\int_{\Omega} z_{i_k} \left(-m_1 - \frac{m_2}{b} z_{i_k} + \frac{w_{i_k}}{1+q\rho_{i_k} w_{i_k}} \right) dx < 0,$$

which is a contradiction, and consequently Eq. (3.10) holds. Then we claim that there exists a positive constant C_9 such that

$$\inf_{\Omega} z_i \ge C_9 \quad \text{for all} \quad i \ge 1. \tag{3.11}$$

If it does not hold, then there exists a subsequence $\{i_k\}_{k=1}^{\infty}$ such that $\lim_{k\to\infty} z_{i_k} = 0$, which implies that $z_{i_k} \to 0$ uniformly on $\overline{\Omega}$ as $k \to \infty$ from Eq. (3.8). Note that $\lim_{k\to\infty} \rho_{i_k} w_{i_k} \to 0$ uniformly on $\overline{\Omega}$ as $k \to \infty$. It follows that, for sufficiently large k,

$$\int_{\Omega} w_{i_k} \left(\frac{rb}{b+kz_{i_k}} - d - a\rho_{i_k} w_{i_k} - \frac{z_{i_k}}{1+q\rho_{i_k} w_{i_k}} \right) dx > 0,$$

which is a contradiction, and consequently Eq. (3.11) holds.

From the above analysis, we see that $\{w_i(x)\}_{i=1}^{\infty}$ and$\{z_i(x)\}_{i=1}^{\infty}$ are bounded in $L^{\infty}(\Omega)$. It follows from the L^p theory that we obtain that $\{w_i(x)\}_{i=1}^{\infty}, \{z_i(x)\}_{i=1}^{\infty}$ are bounded in $W^{2,p}(\Omega)$ for any $p > N$. By virtue of the embedding theorem, we see that $\{w_i(x)\}_{i=1}^{\infty}, \{z_i(x)\}_{i=1}^{\infty}$ are precompact in $C^1(\overline{\Omega})$. Then, there exists a subsequence $\{i_k\}_{k=1}^{\infty}$ and $(\tilde{w}(x), \tilde{z}(x)) \in C^1(\overline{\Omega}) \times C^1(\overline{\Omega})$ such that

$$(w_{i_k}(x), z_{i_k}(x)) \to (\tilde{w}(x), \tilde{z}(x)) \text{ in } C^1(\overline{\Omega}) \text{ as } k \to \infty.$$

Since

$$\begin{aligned} w_{i_k} =& [-d_1\Delta + I]^{-1}\left[w_{i_k} + w_{i_k}\left(\frac{rb}{b + kz_{i_k}} - d - a\rho_{i_k}w_{i_k} - \frac{z_{i_k}}{1 + q\rho_{i_k}w_{i_k}}\right)\right], \\ z_{i_k} =& [-d_2\Delta + I]^{-1}\left[z_{i_k} + z_{i_k}\left(-m_1 - \frac{m_2}{b}z_{i_k} + \frac{w_{i_k}}{1 + q\rho_{i_k}w_{i_k}}\right)\right], \end{aligned} \tag{3.12}$$

and $\lim_{k\to 0}\rho_{i_k}w_{i_k} = 0$ in $C^1(\overline{\Omega})$, we see that $(\tilde{w}(x), \tilde{z}(x))$ is a positive solution of system (3.2) for $\rho = 0$, and it follows from the Schauder theory that

$$(w_{i_k}, z_{i_k}) \to (\tilde{w}(x), \tilde{z}(x)) \text{ in } C^2(\overline{\Omega}) \text{ as } k \to \infty.$$

This completes the proof. □

Now, based on Propositions 3.1 and 3.2, we prove that that system (3.2) has no nonconstant positive solutions for small ρ.

Theorem 3.1. *Assume that $r > d$. Then there exists a positive constant ρ_* such that system* (3.2) *has a unique positive constant solution and has no nonconstant positive solutions for $\rho \in (0, \rho_*)$.*

Proof. By way of contradiction, there exists $\{\rho_i\}_{i=1}^{\infty}$ such that $\lim_{i\to\infty}\rho_i = 0$, and system (3.2) has a nonconstant positive steady state $(w_i(x), z_i(x))$ for $\rho = \rho_i$ $(i = 1, 2, 3, \cdots)$. It follows from Theorems 3.1 and 3.2 that there exists a subsequence $\{i_k\}_{k=1}^{\infty}$ such that $(w_{i_k}(x), z_{i_k}(x)) \to (w_*, z_*)$ in $C^2(\overline{\Omega})$ as $k \to \infty$, where (w_*, z_*) defined as in Theorem 3.1 is the unique positive solution of system (3.2) for $\rho = 0$. Note that 0 is not the eigenvalue with respect to (w_*, z_*) for the corresponding parabolic equations when $\rho = 0$. Then, by virtue of the implicit function theorem, we obtain that there exists $\rho_* > 0$ such that, for $\rho \in (0, \rho_*)$, system (3.2) has a unique positive solution in the neighborhood of (w_*, z_*) in $C^1(\overline{\Omega})$, which is constant. This implies that $(w_{i_k}(x), z_{i_k}(x))$ is constant for sufficiently large k, which is a contradiction. This completes the proof. □

Then we obtain that the original system (3.1) has no nonconstant positive solutions for large conversion rate c.

Corollary 3.1. *Assume that $r > d$. Then there exists a positive constant c_* such that system* (3.1) *has a unique constant positive solution and has no nonconstant positive solutions for $c > c_*$.*

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