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Brief paper

Complete controllability of impulsive stochastic integro-differential systems ☆

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ABSTRACT

This paper is concerned with the controllability of impulsive stochastic integro-differential systems. Sufficient conditions of complete controllability for impulsive stochastic integro-differential systems are obtained by using Schaefer's fixed point theorem. A numerical example is provided to show the effectiveness of the proposed results.

1. Introduction

It is well known that the concept of controllability plays an important role in control theory and engineering. Controllability has been studied extensively in the fields of finite-dimensional nonlinear systems, infinite-dimensional systems (see e.g., Bemporad, Ferrari-Trecate, & Morari, 2000; Li & Rao, 2003; Mahmudov, 2003).

Impulsive systems arise naturally in various fields, such as mechanical systems and biological systems, economics, etc. (see Lakshmikantham, Bainov, & Simeonov, 1989, and the references therein). Impulsive dynamical systems exhibit the continuous evolutions of the states typically described by ordinary differential equations coupled with instantaneous state jumps or impulses. And the presence of impulses implies that the trajectories of the system do not necessarily preserve the basic properties of the non-impulsive dynamical systems. To this end the theory of impulsive differential systems has emerged as an important area of investigation in applied sciences. In the last few years many papers have been published about the controllability of impulsive differential systems. Guan, Qian, and Yu (2002) considered the controllability and observability for a class of time-varying impulsive control systems; Li, Wang, and Zhang (2006) investigated the controllability of the first-order impulsive functional differential systems in Banach space; Xie and Wang (2004) studied the controllability of switched impulsive control systems; Liu and Marquez (2008) discussed the controllability and observability problem for a class of controlled switching impulsive systems; in Sakthivel, Mahmudov, and Kim (2009), sufficient conditions were formulated for the exact controllability of second-order nonlinear impulsive control differential systems.

On the other hand noise is ubiquitous. Systems, both natural and artificial ones, often possess various structures subject to stochastically abrupt changes, which may result from abrupt phenomena such as stochastic failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, etc. More details on stochastic differential equations can be found in the books of Mao (1997) and Oksendal (2003). For linear stochastic system the controllability problem of the form

$$\begin{aligned} dx(t) &= [Ax(t) + Bu(t)]dt + g(t)dw(t), \quad t \in [0, T], \\ x(0) &= x_0, \end{aligned} \tag{1}$$

has been studied by several authors (e.g., Mahmudov, 2001a,b). Here A, B are both $n \times n$ matrices, and $g(\cdot) : [0, T] \to \mathbb{R}^{n\times l}$ for $n, l \in \mathbb{N}$. For nonlinear stochastic systems there are also many results on the control theory, including Mahmudov (2003), Mahmudov and Zorlu (2003), Wang, Ho, Liu, and Liu (2009) and Niu, Ho, and Wang (2007) dealt with the problem of sliding mode control for a class of nonlinear uncertain stochastic systems with Markovian switching. Dong and Sun (2008) gave a detailed discussion on hybrid control for a class of nonlinear stochastic Markovian switching systems.

For the controllability problem there are different methods for various types of nonlinear systems. And the most common

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methods for controllability of stochastic systems as we know are: Picard type iteration (e.g. Balachandran, Karthikeyan, & Kim, 2007), contraction mapping principle (e.g., Mahmudov & Zorlu, 2003), and Lyapunov approach (e.g. Zhao, 2008). However, the complete controllability problem of impulsive stochastic integro-differential systems has not been investigated yet, to the best of our knowledge, although Karthikeyan and Balachandran (2009) and Sakthivel, Mahmudov, and Lee (2009) respectively investigated the controllability of impulsive stochastic control systems by using contraction mapping principle, and Subalakshmi and Balachandran (2009) studied the approximate controllability of nonlinear stochastic impulsive systems in Hilbert spaces by using Nussbaum's fixed point theorem. Based on Schaefer's fixed point theorem, the proposed work in this paper on the complete controllability of integro-differential systems with both noise perturbations and impulsive effects is new in the literature. This problem is important and challenging in both theory and practice, which has motivated us for this study.

In this paper our main aim is to show the complete controllability of the impulsive stochastic integro-differential systems of the form

$$\begin{aligned} \mathrm{d}x(t) &= \left[Ax(t) + F\left(t, x(t), \int_0^t f_1(t,s,x(s))\mathrm{d}s, \int_0^t f_2(t,s,x(s))\mathrm{d}w(s)\right)\right]\mathrm{d}t + Bu(t)\mathrm{d}t + G\left(t, x(t), \int_0^t g_1(t,s,x(s))\mathrm{d}s, \int_0^t g_2(t,s,x(s))\mathrm{d}w(s)\right)\mathrm{d}w(t), \\ &\qquad t \in [0,T],\ t \neq \tau_k,\ k = 1,2,\ldots,m, \\ \Delta x(t) &= I_k(x(t^-)), \quad t = \tau_k,\ k = 1,2,\ldots,m, \\ x(0) &= x_0 \end{aligned} \tag{2}$$

under some basic assumptions via Schaefer's fixed point theorem. Here A and B are both $n \times n$ matrices; and the functions in the equation are:

$F : [0,T] \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$,
$G : [0,T] \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times l}$,
$f_1, g_1 : [0,T] \times [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$,
$f_2, g_2 : [0,T] \times [0,T] \times \mathbb{R}^n \to \mathbb{R}^{n\times l}$;

$\Delta x(t)$ denotes the jump of x at t, i.e.

$\Delta x(t) = x(t^+) - x(t^-) = x(t) - x(t^-)$;

$I_k \in C(\mathbb{R}^n, \mathbb{R}^n)$; The initial value x_0 is a $\mathscr{F}_0$-measurable random variable with $\mathbb{E}\|x_0\|^2 < \infty$; $u(t)$ is a feedback control and w is l-dimensional Wiener process; and $\mathscr{F}_t$ is the filtration generated by $B(s), 0 \le s \le t$.

The system (2) is in a very general form and it covers many possible models with various definitions of f_1, f_2, g_1, g_2. We would like to mention that Balachandran and Karthikeyan (2008) obtained the controllability results of system (2) when $I_k = 0$. The controllability problem with $f_2 = g_1 = g_2 = I_k = 0$ was studied by Sakthivel, Kim, and Mahmudov (2006) and Sakthivel et al. (2009) studied the case $f_1 = f_2 = g_1 = g_2 = 0$. The system (2) with $f_1 = f_2 = g_1 = g_2 = I_k = 0$ was investigated by Mahmudov and Zorlu (2003). However, all the papers listed above obtained the controllability results by using the contraction mapping principle which seems to be restrictive in some cases. In Section 4 an example will illustrate it.

The paper is organized as follows. In Section 2, some basic notations and preliminary facts are recalled. Some lemmas and the results are given in Section 3. And an example in Section 4 is discussed to illustrate the efficiency of the results. Finally, conclusions are given in Section 5.

2. Preliminaries

In this section, we introduce notations, definitions and preliminary facts which are used throughout the paper.

Let $\{\Omega, \mathscr{F}, P\}$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e. right continuous and $\mathscr{F}_0$ containing all P-null sets). $\mathbb{E}(\cdot)$ is the expectation with respect to the measure P. $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega, \mathbb{R}^n)$ is the Banach space of all square integrable and $\mathscr{F}_t$-adapted processes $x(t)$ mapping $[0,T] \times \mathbb{R}^n$ into $\mathbb{R}^n$ equipped with the norm

$$\|x\|_{\mathscr{L}}^2 = \sup_{t\in[0,T]} \mathbb{E}\|x(t)\|^2.$$

For the simplicity of considerations we generally assume that the set of admissible control sets is

$$\mathscr{U}_{\mathrm{ad}} = \mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega, \mathbb{R}^n).$$

Let $\Phi(t) = \exp(At)$. Define the controllability matrix

$$\Psi_s^T = \int_s^T \Phi(T-t)BB^*\Phi^*(T-t)\mathrm{d}t, \quad 0 \le s < t.$$

The controllability operator Π_0^T is the linear transformation from $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega, \mathbb{R}^n)$ to $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega, \mathbb{R}^n)$, associated with (2)

$$\Pi_0^T\{\cdot\} = \int_0^T \Phi(T-t)BB^*\Phi(T-t)\mathbb{E}\{\cdot|\mathscr{F}_t\}\mathrm{d}t.$$

For arbitrary $x_1 \in \mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega, \mathbb{R}^n)$ define by Q the process

$$Q = x_1 - \Phi(T)x_0 - \int_0^T \Phi(T-s)(\tilde{F}x)(s)\mathrm{d}s - \int_0^T \Phi(T-s)(\tilde{G}x)(s)\mathrm{d}w(s) - \sum_{k=1}^m \Phi(T-\tau_k)I_k(x(\tau_k)),$$

where

$$(\tilde{F}x)(t) = F\left(t, x(t), \int_0^t f_1(t,s,x(s))\mathrm{d}s, \int_0^t f_2(t,s,x(s))\mathrm{d}w(s)\right),$$

$$(\tilde{G}x)(t) = G\left(t, x(t), \int_0^t g_1(t,s,x(s))\mathrm{d}s, \int_0^t g_2(t,s,x(s))\mathrm{d}w(s)\right).$$

Our main results are based on the following fixed point theorem of Schaefer which was discussed and proved in Smart (1980).

Lemma 1 (*Schaefer's Theorem*). *Let $(D, \|\cdot\|)$ be a normed space, and let the operator $\mathscr{A} : D \to D$ be completely continuous. Define*

$$S(\mathscr{A}) = \{x \in D : x = \lambda\mathscr{A}x, \lambda \in (0,1)\}.$$

Then either
(1) the set $S(\mathscr{A})$ is unbounded, or
(2) the operator $\mathscr{A}$ has a fixed point in D.

Definition 1. The system (2) is completely controllable on $[0,T]$ if all the points in $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega, \mathbb{R}^n)$ can be reached from x_0 at time T.

Definition 2. A set $M \subset \mathbb{R}^n$ is said to be quasi-equicontinuous in $[0,T]$ if for any $\varepsilon > 0$ there exists $\delta > 0$ such that if $y \in M$, $t_1, t_2 \in (\tau_{k-1}, \tau_k]$, $k \in \mathbb{N}$ and $|t_1 - t_2| < \delta$, then $\|y(t_1) - y(t_2)\| < \varepsilon$.

This together with the standard result (Conway, 1990, pp. 175–176) yields the following necessary and sufficient conditions for relative compactness in $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega, \mathbb{R}^n)$.

Lemma 2. *The set $\Lambda \subset \mathbb{R}^n$ is relatively compact if and only if*
(a) Λ is uniformly bounded, that is, $\|x\|_\Lambda \le B$ for each $x \in \Lambda$ and some $B > 0$.
(b) Λ is quasi-equicontinuous in $[0,T]$.

3. Main results

In this section we discuss the controllability of the stochastic impulsive integro-differential systems (2). For the study of this problem we hence introduce the following hypotheses.
(H_1) The linear control system (1) is completely controllable in $[0, T]$.

By (H_1) there exists a positive constant l_1, such that for $t \in [0, T]$ (Mahmudov, 2001a)

$\|\Phi(t)\|^2 \le l_1$.

(H_2) $(\tilde{F}, \tilde{G})$ are $\mathscr{F}_t$-adapted with respect to t. And for every positive constant k_1, there exists function $q(\cdot) \in L_1(\mathbb{R}^+, \mathbb{R}^+)$ such that for every $t \in [0, T]$,

$$\sup_{\|x\|\le k_1} (\|(\tilde{F}x)(t)\|^2 + \|(\tilde{G}x)(t)\|^2) \le q(t).$$

Furthermore, for every $t \in [0, T]$,

$$\mathbb{E}\int_0^t (\|(\tilde{F}x)(s)\|^2 + \|(\tilde{G}x)(s)\|^2)\mathrm{d}s \quad \text{exists},$$

and

$\mathbb{E}(\|(\tilde{F}x)(t)\|^2 + \|(\tilde{G}x)(t)\|^2)$ exists uniformly for $t \in [0, T]$.

(H_3) $I_k \in C(\mathbb{R}^n, \mathbb{R}^n)$, $k = 1, 2, \ldots, m$ and there exist constants d_k such that $\|I_k(x)\| \le d_k$, $k = 1, 2, \ldots, m$ for each $x \in \mathscr{L}_2^{\mathscr{F}_t}([0, T] \times \Omega, \mathbb{R}^n)$.
(H_4) There exist positive constants M, M_2, l_2 such that

$$M = \max\{\|\Psi_s^t\|^2,\ s \in [0, T]\}, \qquad \|B\| \le M_2,$$

$\|x_1\|^2 + \|x_0\|^2 \le l_2$.

(H_5) There exists a continuous nondecreasing function $\phi : [0, \infty) \to [0, \infty)$ such that $\mathbb{E}\|(\tilde{F}x)(t)\|^2 + \mathbb{E}\|(\tilde{G}x)(t)\|^2 \le a(t)\phi(\mathbb{E}\|x\|^2)$ for all $t \in [0, T]$, $x \in \mathscr{L}_2^{\mathscr{F}_t}([0, T] \times \Omega, \mathbb{R}^n)$, where $a(t) : [0, \infty) \to [0, \infty)$ is an integrable function with

$$M_1 \int_0^T a(s)\mathrm{d}s < \int_c^\infty \frac{\mathrm{d}s}{\phi(s)},$$

where $c = 5l_1l_2 + 25Ml_1l_3(l_2 + l_1l_2 + l_1T\int_0^T \mathbb{E}\|(\tilde{F}x)(s)\|^2\mathrm{d}s + l_1\int_0^T \mathbb{E}\|(\tilde{G}x)(s)\|^2\mathrm{d}s + l_1(\sum_{k=1}^m d_k)^2) + 5l_1(\sum_{k=1}^m d_k)^2$, $M_1 = 5l_1(T+1)$, and l_3 is a positive constant satisfying the following Lemma 3.

Lemma 3 (Mahmudov & Zorlu, 2003). *Assume* (H_1) *holds. Then for every* $z \in \mathscr{L}_2^{\mathscr{F}_t}([0, T] \times \Omega, \mathbb{R}^n)$ *there exists some constant* l_3 *such that*

$$\mathbb{E}\|(\Pi_0^T)^{-1}\|^2 \le l_3 \quad \text{and} \quad \mathbb{E}\|\Pi_0^t z\|^2 \le M\mathbb{E}\|z\|^2.$$

For arbitrary process $x(\cdot) \in \mathscr{L}_2^{\mathscr{F}_t}([0, T] \times \Omega, \mathbb{R}^n)$ define the control

$$u(t) = B^*\Phi^*(T - t)\mathbb{E}\{(\Pi_0^T)^{-1}Q|\mathscr{F}_t\}. \tag{3}$$

In addition we need further hypotheses on $(\tilde{F}, \tilde{G})$, For example, $(\tilde{F}, \tilde{G})$ are Borel-measurable functions satisfying the local Lipschitz condition and the linear growth condition. Under these conditions Liu (2008) showed that there is a unique global solution $x(t, t_0, x_0, u)$ of (2) for any $x_0 \in \mathbb{R}^n$, $t \in [0, T]$, which can be represented in the following form

$$x(t) = \Phi(t)x_0 + \int_0^t \Phi(t - s)[Bu(s) + (\tilde{F}x)(s)]\mathrm{d}s + \int_0^t \Phi(t - s)(\tilde{G}x)(s)\mathrm{d}w(s) + \sum_{0<\tau_k<t} \Phi(t - \tau_k)I_k(x(\tau_k)). \tag{4}$$

Lemma 4. *Assume the operator* Π_0^T *is invertible, then for arbitrary* $x_1 \in \mathscr{L}_2^{\mathscr{F}_t}([0, T] \times \Omega, \mathbb{R}^n)$, *the control* u, *defined by* (3), *transfers the system* (4) *from* x_0 *to* x_1 *at time* T.

Proof. Substituting (3) into (4) we can obtain that

$$\begin{aligned} x(t) &= \Phi(t)x_0 + \int_0^t \Phi(t - s)\Big[BB^*\Phi^*(T - s) \\ &\quad \times \mathbb{E}\left\{(\Pi_0^T)^{-1}Q|\mathscr{F}_s\right\} + (\tilde{F}x)(s)\Big]\mathrm{d}s + \int_0^t \Phi(t - s) \\ &\quad \times (\tilde{G}x)(s)\mathrm{d}w(s) + \sum_{0<\tau_k<t} \Phi(t - \tau_k)I_k(x(\tau_k)) \\ &= \Phi(t)x_0 + \int_0^t \Phi(t - s)[BB^*\Phi^*(t - s)\Phi^*(T - t) \\ &\quad \times \mathbb{E}\{(\Pi_0^T)^{-1}Q|\mathscr{F}_s\} + (\tilde{F}x)(s)]\mathrm{d}s + \int_0^t \Phi(t - s) \\ &\quad \times (\tilde{G}x)(s)\mathrm{d}w(s) + \sum_{0<\tau_k<t} \Phi(t - \tau_k)I_k(x(\tau_k)) \\ &= \Phi(t)x_0 + \Pi_0^t(\Phi^*(T - t)(\Pi_0^T)^{-1}Q) + \int_0^t \Phi(t - s) \\ &\quad \times (\tilde{F}x)(s)\mathrm{d}s + \int_0^t \Phi(t - s)(\tilde{G}x)(s)\mathrm{d}w(s) \\ &\quad + \sum_{0<\tau_k<t} \Phi(t - \tau_k)I_k(x(\tau_k)). \end{aligned} \tag{5}$$

When $t = T$ in (5) it is obvious $x(T) = x_1$. This completes the proof. □

Define the operator $\mathscr{P}$ as

$$\begin{aligned} (\mathscr{P}x)(t) &= \Phi(t)x_0 + \Pi_0^t(\Phi^*(T - t)(\Pi_0^T)^{-1}Q) \\ &\quad + \int_0^t \Phi(t - s)(\tilde{F}x)(s)\mathrm{d}s + \int_0^t \Phi(t - s) \\ &\quad \times (\tilde{G}x)(s)\mathrm{d}w(s) + \sum_{0<\tau_k<t} \Phi(t - \tau_k)I_k(x(\tau_k)). \end{aligned}$$

From Lemma 4 if the operator $\mathscr{P}$ has a fixed point then the system (2) has a solution $x(t, u)$ with respect to $u(\cdot)$. And clearly $x(T, u) = x_1$, then the system (2) is controllable by $u(\cdot)$. Thus the problem to discuss the controllability of the system (2) can be reduced into the existence of the fixed point of $\mathscr{P}$.

Theorem. *Assume the conditions* (H_1)–(H_5) *are satisfied, then the system* (2) *is completely controllable by the control* $u(\cdot)$ *defined by* (3).

Proof. The first step is to obtain *a priori* bound of the set $S(\mathscr{P}) := \{x \in \mathscr{L}_2^{\mathscr{F}_t}([0, T] \times \Omega, \mathbb{R}^n), x = \lambda\mathscr{P}x \text{ for some } \lambda \in (0, 1)\}$.

Let $x \in S(\mathscr{P})$, then $x = \lambda\mathscr{P}(x)$ for some $0 < \lambda < 1$. Thus for each $t \in [0, T]$ we have

$$\begin{aligned} x(t) &= \lambda\Phi(t)x_0 + \lambda\Pi_0^t(\Phi^*(T - t)(\Pi_0^T)^{-1}Q) + \lambda\int_0^t \Phi(t - s) \\ &\quad \times (\tilde{F}x)(s)\mathrm{d}s + \lambda\int_0^t \Phi(t - s)(\tilde{G}x)(s)\mathrm{d}w(s) \\ &\quad + \lambda\sum_{0<\tau_k<t} \Phi(t - \tau_k)I_k(x(\tau_k)), \end{aligned}$$

which implies

$$\begin{aligned} \mathbb{E}\|x(t)\|^2 &< \mathbb{E}\Bigg\|\Phi(t)x_0 + \Pi_0^t(\Phi^*(T - t)(\Pi_0^T)^{-1}Q) \\ &\quad + \int_0^t \Phi(t - s)(\tilde{F}x)(s)\mathrm{d}s + \int_0^t \Phi(t - s)(\tilde{G}x)(s)\mathrm{d}w(s) \\ &\quad + \sum_{0<\tau_k<t} \Phi(t - \tau_k)I_k(x(\tau_k))\Bigg\|^2 \end{aligned}$$

$$\leq 5l_1\mathbb{E}\left(\|x_0\|^2 + Ml_3E\|Q\|^2 + T\int_0^t \|(\tilde{F}x)(s)\|^2 \mathrm{d}s + \int_0^t \|(\tilde{G}x)(s)\|^2 \mathrm{d}s + \left(\sum_{k=1}^m d_k\right)^2\right)$$

$$\leq 5l_1l_2 + 25Ml_1l_3\left(l_2 + l_1l_2 + l_1T\mathbb{E}\int_0^T \|(\tilde{F}x)(s)\|^2 \mathrm{d}s + l_1\mathbb{E}\int_0^T \|(\tilde{G}x)(s)\|^2 \mathrm{d}s + l_1\left(\sum_{k=1}^m d_k\right)^2\right) + 5l_1\left(\sum_{k=1}^m d_k\right)^2 + 5Tl_1\mathbb{E}\int_0^t \|(\tilde{F}x)(s)\|^2 \mathrm{d}s + 5l_1\mathbb{E}\int_0^t \|(\tilde{G}x)(s)\|^2 \mathrm{d}s.$$

We consider the function $\mu(t)$ defined by

$$\mu(t) = \sup\{\mathbb{E}\|x(s)\|^2,\ 0 \leq s \leq t,\ t \in [0, T]\},$$

with the previous inequality, we have for $t \in [0, T]$ that

$$\mu(t) \leq 5l_1l_2 + 25Ml_1l_3\left(l_2 + l_1l_2 + l_1T\mathbb{E}\int_0^T \|(\tilde{F}x)(s)\|^2 \mathrm{d}s + l_1\mathbb{E}\int_0^T \|(\tilde{G}x)(s)\|^2 \mathrm{d}s + l_1\left(\sum_{k=1}^m d_k\right)^2\right) + 5l_1\left(\sum_{k=1}^m d_k\right)^2 + 5Tl_1\mathbb{E}\int_0^t \|(\tilde{F}x)(s)\|^2 \mathrm{d}s + 5l_1\mathbb{E}\int_0^t \|(\tilde{G}x)(s)\|^2 \mathrm{d}s.$$

Take the right-hand side of the inequality above as $v(t)$ then we get

$$c = v(0) = 5l_1l_2 + 25Ml_1l_3\left(l_2 + l_1l_2 + l_1T\mathbb{E}\int_0^T \|(\tilde{F}x)(s)\|^2 \mathrm{d}s + l_1\mathbb{E}\int_0^T \|(\tilde{G}x)(s)\|^2 \mathrm{d}s + l_1\left(\sum_{k=1}^m d_k\right)^2\right) + 5l_1\left(\sum_{k=1}^m d_k\right)^2,$$

$$\mu(t) \leq v(t), \quad t \in [0, T],$$

and by (H_5) we can obtain

$$v'(t) = 5l_1(T\mathbb{E}\|(\tilde{F})(x)(t)\|^2 + \mathbb{E}\|\tilde{G}(x)(t)\|^2) \leq 5l_1(T+1)a(t)\phi(\mathbb{E}\|x(t)\|^2) \leq 5l_1(T+1)a(t)\phi(v(t)).$$

That implies that for each $t \in [0, T]$,

$$\int_{v(0)}^{v(t)} \frac{\mathrm{d}s}{\phi(s)} \leq \int_0^t M_1a(s)\mathrm{d}s \leq \int_0^T M_1a(s)\mathrm{d}s < \int_c^\infty \frac{\mathrm{d}s}{\phi(s)}.$$

This inequality implies there exists a constant k such that $v(t) \leq k, t \in [0, T]$ and hence $\mu(t) \leq k, t \in [0, T]$. Since for every $t \in [0, T]$, $\|x\|^2_{\mathscr{L}} \leq \mu(t)$, we have

$$\|x\|^2_{\mathscr{L}} = \sup_{t\in[0,T]} \mathbb{E}\|x(t)\|^2 \leq k$$

and the set $S(\mathscr{P})$ is bounded.

Let $x \in B_q = \{x \in \mathscr{L}_2^{\mathscr{F}_t}([0, T] \times \Omega, \mathbb{R}^n), \|x\| \leq q\}, t_1, t_2 \in [0, T]$ with $t_1 \leq t_2$. We will show $\mathscr{P}$ maps bounded sets B_q into equicontinuous sets.

$$\mathbb{E}\|(\mathscr{P}x)(t_1) - (\mathscr{P}x)(t_2)\|^2$$

$$= \mathbb{E}\Bigg\|(\Phi(t_1) - \Phi(t_2))x_0 + \Pi_0^{t_1}(\Phi^*(T-t_1)(\Pi_0^T)^{-1}Q) + \int_0^{t_1}\Phi(t_1-s)(\tilde{F}x)(s)\mathrm{d}s + \int_0^{t_1}\Phi(t_2-s)(\tilde{G}x)(s)\mathrm{d}w(s) + \sum_{0<\tau_k<t_1}\Phi(t_1-\tau_k)I_k(x(\tau_k)) - \Pi_0^{t_2}(\Phi^*(T-t_2)(\Pi_0^T)^{-1}\cdot Q) - \int_0^{t_2}\Phi(t_2-s)(\tilde{F}x)(s)\mathrm{d}s - \int_0^{t_2}\Phi(t_2-s)\cdot(\tilde{G}x)(s)\mathrm{d}w(s) - \sum_{0<\tau_k<t_2}\Phi(t_2-\tau_k)I_k(x(\tau_k))\Bigg\|^2$$

$$= \mathbb{E}\Bigg\|(\Phi(t_1) - \Phi(t_2))x_0 + \int_0^{t_1}(\Phi(t_1-s) - \Phi(t_2-s))(\tilde{F}x)(s)\mathrm{d}s - \int_{t_1}^{t_2}\Phi(t_2-s)(\tilde{F}x)(s)\mathrm{d}s + \int_0^{t_1}(\Phi(t_1-s) - \Phi(t_2-s)) \times (\tilde{G}x)(s)\mathrm{d}w(s) - \int_{t_1}^{t_2}\Phi(t_2-s)(\tilde{G}x)(s)\mathrm{d}w(s) + \sum_{0<t<t_1}(\Phi(t_1-s) - \Phi(t_2-s))I_k(x(\tau_k)) + \int_0^{t_1}(\Phi(t_1-s) - \Phi(t_2-s))BB^*\Phi^*(T-s)\mathbb{E}\{Q|\mathscr{F}_s\}\mathrm{d}s - \int_{t_1}^{t_2}\Phi(t_2-s)BB^*\Phi^*(T-s)\mathbb{E}\{Q|\mathscr{F}_s\}\mathrm{d}s - \sum_{t_1\leq t<t_2}\Phi(t_2-s)I_k(x(\tau_k))\Bigg\|^2$$

$$\leq 9\mathbb{E}\|(\Phi(t_1) - \Phi(t_2))x_0\|^2 + 9\mathbb{E}\left\|\int_0^{t_1}(\Phi(t_1-s) - \Phi(t_2-s)) \times (\tilde{F}x)(s)\mathrm{d}s\right\|^2 + 9\mathbb{E}\left\|\int_{t_1}^{t_2}\Phi(t_2-s)\cdot(\tilde{F}x)(s)\mathrm{d}s\right\|^2 + 9\mathbb{E}\left\|\int_0^{t_1}(\Phi(t_1-s) - \Phi(t_2-s))\cdot(\tilde{G}x)(s)\mathrm{d}w(s)\right\|^2 + 9\mathbb{E}\left\|\int_{t_1}^{t_2}\Phi(t_2-s)(\tilde{G}x)(s)\mathrm{d}w\right\|^2 + 9\mathbb{E}\left\|\sum_{0<t<t_1}(\Phi(t_1-s) - \Phi(t_2-s))I_k(x(\tau_k))\right\|^2 + 9\mathbb{E}\left\|\int_0^{t_1}(\Phi(t_1-s) - \Phi(t_2-s))BB^*\Phi^*(T-s)\mathbb{E}\{Q|\mathscr{F}_s\}\mathrm{d}s\right\|^2 + 9\mathbb{E}\left\|\int_{t_1}^{t_2}\Phi(t_2-s)BB^*\Phi^*(T-s)\mathbb{E}\{Q|\mathscr{F}_s\}\mathrm{d}s\right\|^2 + 9E\left\|\sum_{t_1\leq t<t_2}\Phi(t_2-s)I_k(x(\tau_k))\right\|^2$$

$$\leq 9\mathbb{E}\|(\Phi(t_1) - \Phi(t_2))x_0\|^2 + 9\int_0^{t_1}\|\Phi(t_1-s) - \Phi(t_2-s)\|^2\mathrm{d}s\int_0^{t_1}q(s)\mathrm{d}s + 9l_1T\int_{t_1}^{t_2}q(s)\mathrm{d}s + 9\int_0^{t_1}\|\Phi(t_1-s) - \Phi(t_2-s)\|^2q(s)\mathrm{d}s$$

$$+9l_1\int_{t_1}^{t_2}q(s)\mathrm{d}s+9\left(\sum_{0<t<t_1}\|\Phi(t_1-s)-\Phi(t_2-s)\|d_k\right)^2$$
$$+45M_2^2l_1\Bigg(l_2+l_1T\int_0^T q(s)\mathrm{d}s+l_1l_2+l_1\int_0^T q(s)\mathrm{d}s$$
$$+l_1\left(\sum_{t=0}^m d_k\right)^2\Bigg)\mathbb{E}\int_0^{t_1}\|\Phi(t_1-s)-\Phi(t_2-s)\|^2\mathrm{d}s$$
$$+45M_2^2l_1\Bigg(l_2+l_1l_2+Tl_1\int_0^T q(s)\mathrm{d}s+l_1\int_0^T q(s)\mathrm{d}s$$
$$+l_1\left(\sum_{t=0}^m d_k\right)^2\Bigg)(t_2-t_1)+l_1\left(\sum_{t_1\le t<t_2}d_k\right)^2.$$

By (H_2) the right-hand side is independent of $x\in B_q$ and tends to zero as $t_2\to t_1$. This proves the equicontinuous case of $t_2>t_1$, and the equicontinuous case of $t_2<t_1$ can be also obtained using the same method. Thus $\mathscr{P}$ maps bounded set into equicontinuous family of functions.

Next we show $\mathscr{P}$ is an compact operator. It suffices to show the closure of $\mathscr{P}B_q$ is compact since compact operator is a linear operator such that the image of any bounded sets is a relatively compact set.

Let $0\le t\le T$ be fixed and ε a real number satisfying $0<\varepsilon<t$. For $x\in B_q$ we define

$$(\mathscr{P}_\varepsilon x)(t)=\Phi(t)x_0+\Pi_0^{t-\varepsilon}(\Phi^*(T-t)(\Pi_0^T)^{-1}Q)+\int_0^{t-\varepsilon}\Phi(t-s)(\tilde{F}x)(s)\mathrm{d}s+\int_0^{t-\varepsilon}\Phi(t-s)\times(\tilde{G}x)(s)\mathrm{d}w(s)+\sum_{0<\tau_k<t-\varepsilon}\Phi(t-\tau_k)I_k(x(\tau_k)).$$

Note that using the same methods in the procedure above we can obtain the quasi-equicontinuous and uniform bounded property of $\mathscr{P}_\varepsilon$, which implies, by the Lemma 2, the set $\{(\mathscr{P}_\varepsilon x)(t), x\in B_q\}$ is relatively compact in $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega,\mathbb{R}^n)$ for every $0<\varepsilon<t$. Moreover, for every $x\in B_q$

$$\|(\mathscr{P}x)(t)-(\mathscr{P}_\varepsilon x)(t)\|\to 0\quad\text{as }\varepsilon\to 0,$$

therefore the relatively compact sets $\{(\mathscr{P}_\varepsilon x)(t), x\in B_q\}$ are arbitrarily close to the set $(\{\mathscr{P}x)(t), x\in B_q\}$. Hence $(\{\mathscr{P}x)(t), x\in B_q\}$ is compact in $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega,\mathbb{R}^n)$ by the Ascoli–Arzela Theorem.

It remains to show $\mathscr{P}$ is continuous. Let x_n be a sequence in $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega,\mathbb{R}^n)$ such that $\|x-x_n\|\to 0$ as $n\to\infty$. Then there is an integer r such that $\|x_n\|\le r$ for all n and $t\in[0,T]$ so $x,x_n\in B_r$. Denote

$$\tilde{F}_1(t)=(\tilde{F}x)(t)-(\tilde{F}x_n)(t),\qquad \tilde{G}_1(t)=(\tilde{G}x)(t)-(\tilde{G}x_n)(t),$$

and

$$\tilde{I}_k(x,x_n)=I_k(x(\tau_k))-I_k(x_n(\tau_k)).$$

It can be derived directly from (H_2) that, for each $t\in[0,T]$, $\tilde{F}_1(t)\to 0$, $\tilde{G}_1(t)\to 0$, $\|\tilde{F}_1\|^2+\|\tilde{G}_1\|^2\le 2q(t)$.

$$\mathbb{E}\|\mathscr{P}x_n-\mathscr{P}x\|^2$$
$$=\mathbb{E}\Bigg\|\Pi_0^t\Bigg[\Phi^*(T-t)(\Pi_0^T)^{-1}\Bigg(\int_0^T\Phi(T-s)\tilde{F}_1(s)\mathrm{d}s+\int_0^T\Phi(T-s)\tilde{G}_1(s)\mathrm{d}w(s)+\sum_{t=0}^m\Phi(T-\tau_k)\tilde{I}_k(x,x_n)\Bigg)\Bigg]$$
$$+\int_0^t\Phi(t-s)\tilde{F}_1(s)\mathrm{d}s+\int_0^t\Phi(t-s)\tilde{G}_1(s)\mathrm{d}w(s)+\sum_{0<\tau_k<t}\Phi(t-\tau_k)\tilde{I}_k(x,x_n)\Bigg\|^2$$
$$\le 4\mathbb{E}\Bigg\|\Pi_0^t\Bigg[\Phi^*(T-t)(\Pi_0^T)^{-1}\Bigg(\int_0^T\Phi(T-s)\tilde{F}_1(s)\mathrm{d}s+\int_0^T\Phi(T-s)\tilde{G}_1(s)\mathrm{d}w(s)+\sum_{k=1}^m\Phi(T-\tau_k)\tilde{I}_k(x,x_n)\Bigg)\Bigg]\Bigg\|^2$$
$$+4\mathbb{E}\left\|\int_0^t\Phi(t-s)\tilde{F}_1(s)\mathrm{d}s\right\|^2+4\mathbb{E}\left\|\int_0^t\Phi(t-s)\tilde{G}_1(s)\mathrm{d}w(s)\right\|^2+4\mathbb{E}\left\|\sum_{0<\tau_k<t}\Phi(t-\tau_k)\tilde{I}_k(x,x_n)\right\|^2$$
$$\le 12Ml_1^2l_3\mathbb{E}\left(\int_0^T(T\|\tilde{F}_1(s)\|^2+\|\tilde{G}_1(s)\|^2)\mathrm{d}s+\sum_{t=0}^m\|\tilde{I}_k(x,x_n)\|^2\right)$$
$$+4l_1\mathbb{E}\left[\int_0^t(T\|\tilde{F}_1(s)\|^2+\|\tilde{G}_1(s)\|^2)\mathrm{d}s+\sum_{0<\tau_k<t}\|\tilde{I}_k(x,x_n)\|^2\right].$$

By (H_2), (H_3) and and also I_k is continuous, we obtain that the previous inequality implies the continuousness of the operator $\mathscr{P}$ by using the dominated convergence theorem.

Consequently by Lemma 1 the operator $\mathscr{P}$ has a fixed point in $\mathscr{L}_2^{\mathscr{F}_t}([0,T]\times\Omega,\mathbb{R}^n)$. That means that any fixed point of $\mathscr{P}$ is a solution of system (2) about u on $[0,T]$. Hence the system (2) is completely controllable on $[0,T]$. This completes the proof. □

4. A numerical example

Consider the following two-dimensional impulsive integro-differential stochastic system in the form of (2), where

$$A=\begin{pmatrix}-1&1\\-1&-1\end{pmatrix},\qquad B=\begin{pmatrix}1&0\\0&1\end{pmatrix},\tag{6}$$

$F=(F_1,F_2)^T$ and

$$F_1=\frac{1+\int_0^t\sin x_1\mathrm{d}s+\int_0^t\cos x_1\mathrm{d}w}{\sqrt{1+|x_1|}},$$
$$F_2=\frac{1+\int_0^t\sin x_2\mathrm{d}s+\int_0^t\cos x_2\mathrm{d}w}{\sqrt{1+|x_2|}}$$

$G=(g_{ij})_{2\times2}$, with $g_{12}=g_{21}=0$, $g_{11}=\mathrm{e}^{-t}|x_1+1|+\left|\int_0^t\arctan x_1\mathrm{d}s\right|+\left|\int_0^t\sin x_1\mathrm{d}w\right|$, $g_{22}=\mathrm{e}^{-t}|x_2+1|+|\int_0^t\arctan x_2\mathrm{d}s|+\left|\int_0^t\sin x_2\mathrm{d}w\right|$,

$$I_k=\operatorname{diag}\left(-1+\mathrm{e}^{-2-\frac{1}{2(k+1)}},-1+\mathrm{e}^{-2-\frac{1}{2(k+1)}}\right)\tag{7}$$

for $x=(x_1,x_2)$ with the initial value x_0 and final point $x_T\in\mathbb{R}^2$. For this system the controllability matrix is

$$\Psi_s^t=\frac{1}{2}\begin{pmatrix}1-\mathrm{e}^{-2(t-s)}&0\\0&1-\mathrm{e}^{-2(t-s)}\end{pmatrix}>0,\quad t>s.$$

That is to say the corresponding linear system is completely controllable (Mahmudov, 2001a). Moreover,

$$\mathbb{E}\|(\tilde{F}x)(t)\|^2 \le 3\mathbb{E}\left(\frac{1+t+t^2}{1+|x_1|}+\frac{1+t+t^2}{1+|x_2|}\right) \le 6(1+t+t^2), \quad (8)$$

and

$$\mathbb{E}\|(\tilde{G}x)(t)\|^2 \le 3\mathbb{E}(\mathrm{e}^{-2t}(x_1+1)^2+6t^2+2t+\mathrm{e}^{-2t}(x_2+1)^2) \le 6(1+t+3t^2)(\mathbb{E}\|x\|^2+2). \quad (9)$$

Therefore the two inequalities above imply that

$$\mathbb{E}(\|(\tilde{F}x)(t)\|^2+\|(\tilde{G}x)(t)\|^2) \le 6(1+t+3t^2)(\mathbb{E}\|x\|^2+3).$$

It can be easily seen that $\tilde{F}$, $\tilde{G}$, I_k satisfy the hypothesis (H_2)–(H_5) on an arbitrary finite interval. Hence the systems is completely controllable on $[0, T]$, $T < \infty$.

However, when considering the system (6) reduced into the corresponding systems discussed, respectively, in Balachandran and Karthikeyan (2008), Mahmudov and Zorlu (2003), Sakthivel, Kim, and Mahmudov (2006) and Sakthivel et al. (2009), the conditions listed in those papers are invalid, although the systems they studied are special cases of the system (6). In addition, if the systems in Karthikeyan and Balachandran (2009) are reduced to (6), the results there cannot work either.

5. Conclusions

The controllability of impulsive stochastic integro-differential systems has been investigated in this paper. By defining an operator on a proper function space, the controllability problem has been transformed into the existence of solution to the operator equation. Based on Schaefer's fixed point theorem, sufficient conditions of complete controllability for the system have been obtained. To illustrate the obtained results, a numerical example has been analyzed.

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References

Balachandran, K., & Karthikeyan, S. (2008). Controllability of nonlinear Ito type stochastic integrodifferential systems. *Journal of the Franklin Institute*, *345*(4), 382–391.
Balachandran, K., Karthikeyan, S., & Kim, J. H. (2007). Controllability of semilinear stochastic integrodifferential systems. *Kybernetika*, *43*(1), 31–44.
Bemporad, A., Ferrari-Trecate, G., & Morari, M. (2000). Observability and controllability of piecewise affine and hybrid systems. *IEEE Transactions on Automatic Control*, *45*(10), 1864–1876.
Conway, J. B. (1990). *A course in functional analysis*. New York: Springer-Verlag.
Dong, Y. W., & Sun, J. T. (2008). On hybrid control of a class of stochastic non-linear Markovian switching systems. *Automatica*, *44*(4), 990–995.
Guan, Z. H., Qian, T. H., & Yu, X. H. (2002). On controllability and observability for a class of impulsive systems. *Systems & Control Letters*, *47*(3), 247–257.
Karthikeyan, S., & Balachandran, K. (2009). Controllability of nonlinear stochastic neutral impulsive systems. *Nonlinear Analysis. Hybrid Systems*, *3*(3), 266–276.
Lakshmikantham, V., Bainov, D. D., & Simeonov, P. S. (1989). *Theory of impulsive differential equations*. Singapore: World Scientific.
Li, T. T., & Rao, B. P. (2003). Exact boundary controllability for quasi-linear hyperbolic systems. *SIAM Journal on Control and Optimization*, *41*(6), 1748–1755.
Li, M. L., Wang, M. S., & Zhang, F. Q. (2006). Controllability of impulsive functional differential systems in Banach spaces. *Chaos, Solitons & Fractals*, *29*(1), 175–181.
Liu, B. (2008). Stability of solutions for stochastic impulsive systems via comparison approach. *IEEE Transactions on Automatic Control*, *53*(9), 2128–2133.
Liu, B., & Marquez, H. J. (2008). Controllability and observability for a class of controlled switching impulsive systems. *IEEE Transactions on Automatic Control*, *53*(10), 2360–2366.
Mahmudov, N. I. (2001a). Controllability of linear stochastic systems. *IEEE Transactions on Automatic Control*, *46*(5), 724–731.
Mahmudov, N. I. (2001b). Controllability of linear stochastic systems in Hilbert spaces. *Journal of Mathematical Analysis and Applications*, *259*(1), 64–82.
Mahmudov, N. I. (2003). Approximate controllability of semilinear deterministic and stochastic evolution equations in abstract spaces. *SIAM Journal on Control and Optimization*, *42*(5), 1604–1622.
Mahmudov, N. I., & Zorlu, S. (2003). Controllability of non-linear stochastic systems. *International Journal of Control*, *76*(2), 95–104.
Mao, X. (1997). *Stochastic differential equations and applications*. Chichester, UK: Horwood.
Niu, Y. G., Ho, D. W. C., & Wang, X. Y. (2007). Sliding mode control for Ito stochastic systems with Markovian switching. *Automatica*, *43*(10), 1784–1790.
Oksendal, B. (2003). *Stochastic differential equations* (6th ed.). Springer-Verlag.
Sakthivel, R., Kim, J. H., & Mahmudov, N. I. (2006). On controllability of nonlinear stochastic systems. *Reports on Mathematical Physics*, *58*(3), 433–443.
Sakthivel, R., Mahmudov, N. I., & Kim, J. H. (2009). On controllability of second order nonlinear impulsive differential systems. *Nonlinear Analysis*, *71*, 45–52.
Sakthivel, R., Mahmudov, N. I., & Lee, S. G. (2009). Controllability of non-linear impulsive stochastic systems. *International Journal of Control*, *82*(5), 801–807.
Smart, D. R. (1980). *Fixed point theorem*. Cambridge: Cambridge Univ. Press.
Subalakshmi, R., & Balachandran, K. (2009). Approximate controllability of nonlinear stochastic impulsive integrodifferential systems in hilbert spaces. *Chaos, Solitons & Fractals*, *42*(4), 2035–2046.
Wang, Z. D., Ho, D. W. C., Liu, Y. R., & Liu, X. H. (2009). Robust H_∞ control for a class of nonlinear discrete time-delay stochastic systems with missing measurements. *Automatica*, *45*(3), 684–691.
Xie, G. M., & Wang, L. (2004). Necessary and sufficient conditions for controllability and observability of switched impulsive control systems. *IEEE Transactions on Automatic Control*, *49*(6), 960–966.
Zhao, P. (2008). Practical stability, controllability and optimal control of stochastic Markovian jump systems with time-delays. *Automatica*, *44*(12), 3120–3125.

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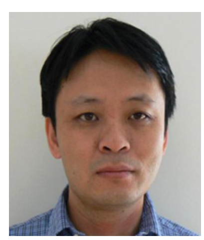

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