

Dimensions:

Dimension is the unit of certain type of physical quantity.

Example: the dimension of variable t (time) is second or hour, the dimension of variable L (length) is meter or foot, and the dimension of parameter N (carrying capacity) is million people.

Dimensionless: A number is dimensionless if it is just a number, it is not a measurement of any type of physical quantity.

Example: Quotient of N (carrying capacity) and P (the population variable) is a dimensionless quantity.

Every variable or parameter in the differential equation has a dimension or is dimensionless.

Nondimensionalization:

Nondimensionalization is a process of changing variables by scaling so that the new variables are dimensionless, and it leads to a simpler form of the equation with fewer parameters.

Step 1: List all variables, parameters and their dimensions.

Step 2: Take each variable and create a new variable by dividing by the combination of parameters that has the same dimension in order to create a dimensionless variable. (Usually there is more than one way to do this.

Step 3: Calculate and simplify the new equation via change of variables.

Step 4: Introduce new parameters.

Example 1:

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{N}\right) \left(\frac{P}{M} - 1\right). \quad (1)$$

Variable	Dimension	Parameter	Dimension
t	τ	k	
P	ρ	M	
		N	

Example 2:

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{N}\right) - \frac{BP^2}{A^2 + P^2}, \quad P(0) = P_0. \quad (2)$$

Variable	Dimension	Parameter	Dimension
t	τ	k	$1/\tau$
P	ρ	N	ρ
		A	ρ
		B	ρ/τ
		P_0	ρ