

# Assignment 2

Math 345, Prof. Shi

Due: Friday, Sept 21 (5pm)

1. Consider a population that disperses between three habitats; habitat 1 that is of very high quality (paradiso), habitat 2 that is of intermediate quality (purgatory), and habitat 3 that is of very low quality (inferno). The populations are censused prior to the adults reproducing. In all habitats, all females produce two daughters and then die. The fraction of daughters surviving in patches 1, 2, and 3 are 0.75 (not quite paradise), 0.5, and 0.1, respectively. After surviving, a constant fraction of progeny  $d$  disperse from their current habitat and go with equal likelihood to either of the two other habitats.
  - (a) Write down a linear difference equation model (or matrix model) for the females in this population.
  - (b) Determine what happens after one generation if initially there were 100 females in each habitat and  $d = 0.2$ .
  - (c) Use Matlab to simulate the solutions for 10 generations with initial condition (100, 200, 300) for  $d = 0.2$  and  $d = 0.8$ .
  - (d) Use Matlab to solve the dominate eigenvalue for  $d = 0.2$  and  $d = 0.8$ .
  - (e) (extra 1 point) Analytically determine the  $d$ -value  $d_1$  such that  $\lambda_1(d_1) = 1$  which is the threshold value for extinction/growth. (Hint: from part (d),  $0.2 < d_1 \ll 0.8$ )
2. (from Page 62 problem 3). Consider the Ricker equation  $N_{n+1} = \alpha N_n e^{-\beta N_n}$ . In this equation,  $\alpha$  is the maximal growth rate, and  $\beta$  is the inhibition of growth caused by overpopulation.
  - (a) Show that the equation has two nonnegative steady state solutions  $N_* = 0$  (for all  $\alpha > 0$ ) and  $N^* = \ln \alpha / \beta$  (for  $\alpha > 1$ ).
  - (b) Show that  $N_* = 0$  is unstable, and  $N^*$  is stable if  $1 < \alpha < e^2 (\approx 7.39)$ .
  - (c) Let  $\beta = 1$  and  $\alpha = 8$ . Use Matlab to generate and plot the first 20 terms of the solution for Ricker equation with  $N_1 = 2$ . How do you describe the behavior of the solutions?
3. (a) Use Matlab to generate and plot the first 30 terms of the solutions to the difference equation
$$x_{n+1} = rx_n(1 - x_n), x_0 = 0.1$$
with  $r = 0.9, 1.5, 3, 4$ .
  - (b) Use Matlab to draw the cobweb maps for each case above.
4. Find the equilibria of  $x_{n+1} = rx_n(1 - x_n)$  for  $r = 1.5$  and  $r = 4$ , and determine the stability of each equilibrium.