A Model for Blue Crab Population in the Chesapeake Bay

Tim Becker[1], Rom Lipcius[2], Leah Shaw[3], Junping Shi[1]

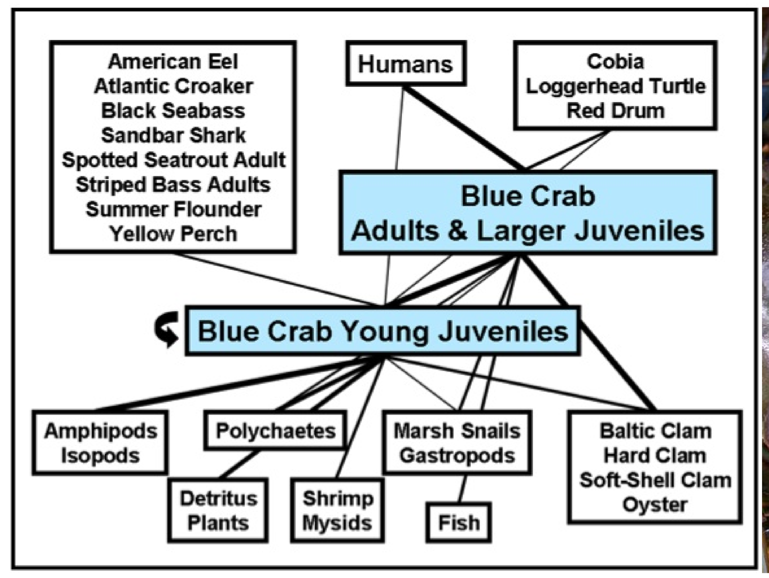
1. Department of Mathematics, The College of William and Mary, Williamsburg, Virginia, 23187-8795
2. Virginia Institute of Marine Science, The College of William and Mary, Gloucester Point, Virginia, 23062
3. Department of Applied Science, The College of William and Mary, Williamsburg, Virginia, 23187-8795

Abstract

The Blue Crab is a cannibalistic predator and partakes in intraguild predation. With this in mind, we model the population of the Blue Crab in the Chesapeake Bay by using differential equations. The differential equations describe the overall biomass per square meter of the adult blue crab, juvenile blue crab, and the resource. We study the effects of cannibalism, as well as the effect of the fisheries on the population dynamics of the system.

Blue Crab–Intraguild Predation

The blue crab partakes in intraguild predation, which is a subset of omnivory. Omnivory is commonly defined as predation over more than one trophic level[1]. In this case, we have the blue crab, which eats both juvenile blue crabs as well as the bivalves and clams that juvenile blue crabs eat[2](pg. 592). In accordance with intraguild predation, the blue crab is inherently cannibalistic[2](pg. 620). Thus, in terms of intraguild predation, the predator is the blue crab adult, while the prey is the blue crab juvenile.



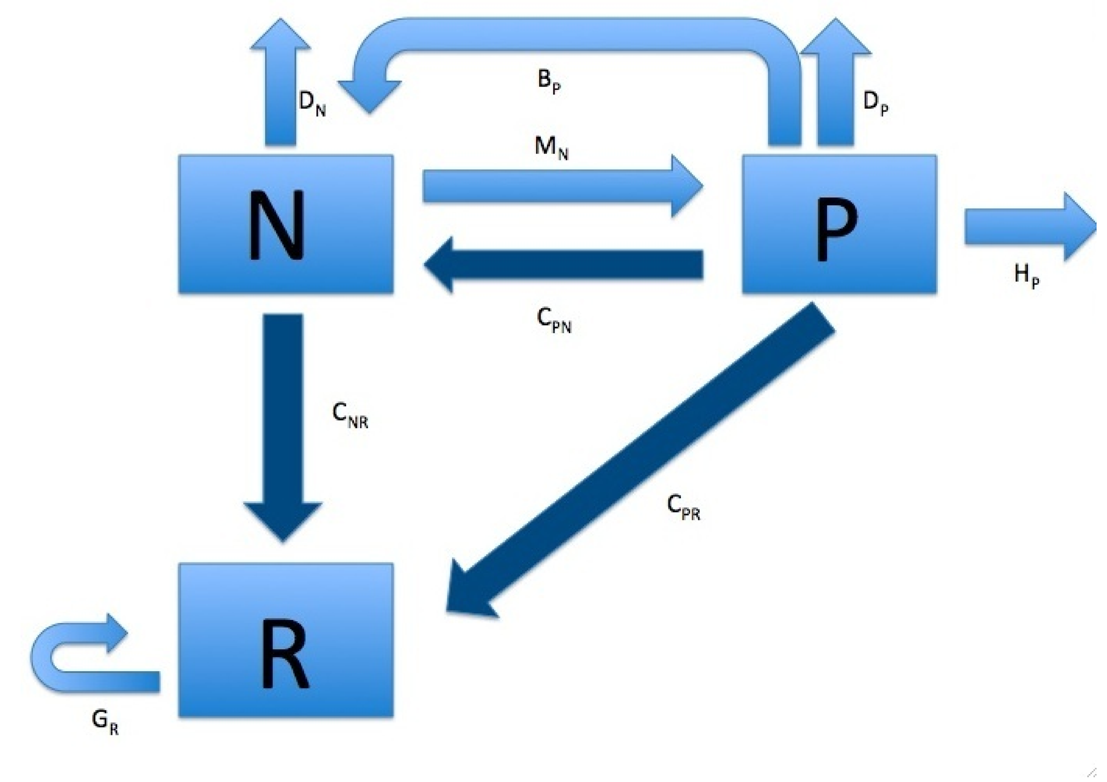
Model Analysis

The Blue Crab is a cannibalistic predator, but one that competes with its offspring for food as well. Accordingly, the model for the Blue Crab must accurately represent the intraguild predation that is present, as well as the cannibalism inherent in the Blue Crab population. We have the following model of the cannibalistic intraguild predation modified from the one of Verdy and Amarasekere [3] as such:

$$
\begin{aligned}
\frac{dR}{dt} &= \underbrace{rR\left(1-\frac{R}{k}\right)}_{G_R} - \underbrace{\frac{aRN}{ahR+1}}_{C_{NR}} - \underbrace{\frac{a'RP}{a'h'R+a''h''N+1}}_{C_{PR}},\\
\frac{dN}{dt} &= \underbrace{\frac{nP}{1+n'P}}_{B_P} + \underbrace{\frac{baRN}{ahR+1}}_{C_{NR}} - \underbrace{eN}_{M_N} - \underbrace{mN}_{D_N} - \underbrace{\frac{a''NP}{a'h'R+a''h''N+1}}_{C_{PN}},\\
\frac{dP}{dt} &= \underbrace{eN}_{M_N} + \underbrace{\frac{b'a'RP}{a'h'R+a''h''N+1}}_{C_{PR}} + \underbrace{\frac{b''a''NP}{a'h'R+a''h''N+1}}_{C_{PN}} - \underbrace{m'P}_{D_P} - \underbrace{fP}_{H_P},
\end{aligned} \tag{1}
$$

where R is the resource, N is the prey, and P is the predator.

The following diagram describes the interaction of the terms of the system:



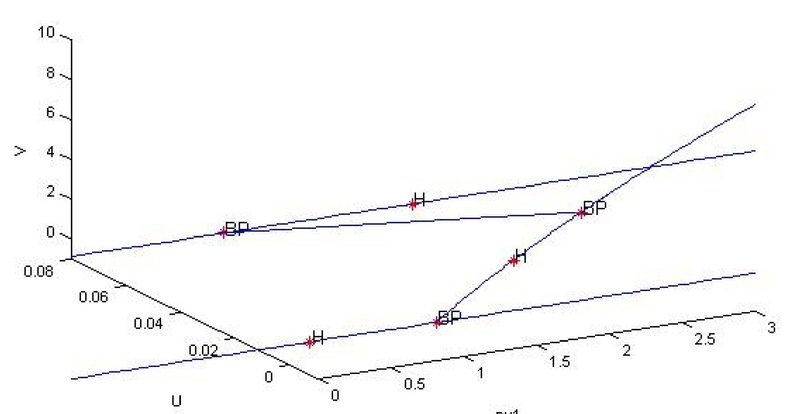
After non-dimensionalization, we have the new dimensionless system:

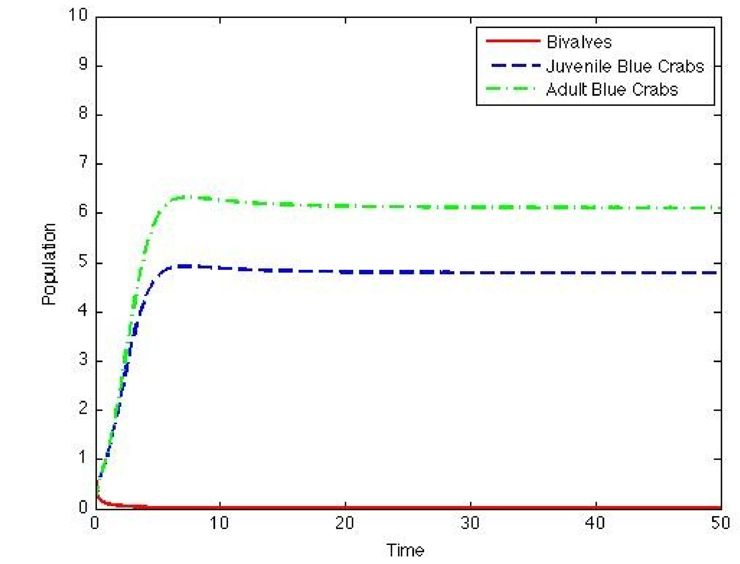
$$
\begin{aligned}
\frac{dU}{ds} &= U\left(1-\frac{U}{\kappa}\right) - \frac{\gamma_1 UV}{U+1} - \frac{\lambda UW}{\theta U+\delta\psi V+1},\\
\frac{dV}{ds} &= \frac{\nu_1 W}{1+\nu_2 W} + \frac{\gamma_2 UV}{U+1} - \mu_1 V - \frac{\delta VW}{\theta U+\delta\psi V+1},\\
\frac{dW}{ds} &= \epsilon V - \mu_2 W + \frac{\gamma_3 UW+\delta\gamma_4 VW}{\theta U+\delta\psi V+1},
\end{aligned} \tag{2}
$$

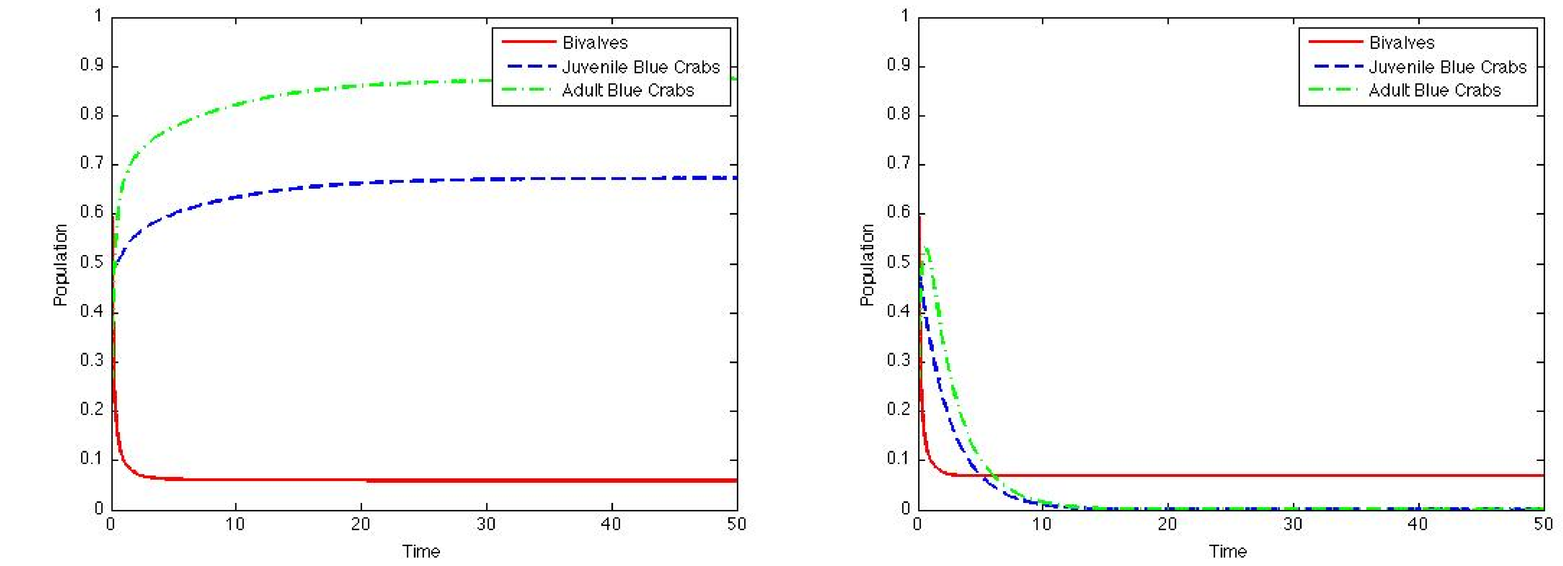
Equilibrium and Stability Analysis

System (2) has three non-negative equilibria with at least one component being zero: $(0,0,0)$, $(\kappa,0,0)$, and $(0,V^*,W^*)$. The trivial equilibrium, $(0,0,0)$ is a saddle point, and, as such, is unstable. Using ν_1 as a bifurcation parameter, we see that the semi-trivial equilibrium $(\kappa,0,0)$ is locally stable when $\nu_1 < \frac{(\frac{\gamma_2\kappa}{\kappa+1}-\mu_1)(\frac{\gamma_3\kappa}{\theta\kappa+1}-\mu_2)}{\epsilon} \equiv \nu_1^*$, and is unstable when $\nu_1 > \nu_1^*$. This is the bifurcation parameter at which the positive solution bifurcates from $(\kappa,0,0)$. We have another bifurcation parameter, $\nu_1^{\#} \equiv \frac{\mu_1\mu_2}{\epsilon}$ when $(0,U^*,V^*)$ bifurcates from the trivial equilibrium. There is a third bifurcation parameter, $\tilde{\nu}_1 > \nu_1^{\#}$, when $(0,V^*,W^*)$ becomes stable.

Numerical Simulations







Parameter used: $\gamma_1 = 0.14$, $\lambda = 0.07$, $\theta = 1$, $\psi = 0.1$, $\kappa = 0.07$, $\nu_2 = 0.1$, $\gamma_2 = 2$, $\delta = 0.07$, $\mu_1 = 1.25$, $\epsilon = 4$, $\gamma_3 = 2$, $\mu_2 = 3.2$, $\gamma_4 = 0.2$.
Initial value: $(U,V,W) = (0.8, 0.5, 0.2)$. Here our bifurcation value is $\nu_1^* = .8587$, $\nu_1^{\#} = 1$, and $\tilde{\nu}_1 = 2.001$
(Left Panel): $\nu_1 = 1$, persistence of blue crab;
(Right Panel): $\nu_1 = .5$, extinction of blue crab;
(Previous Column-Right Panel): $\nu_1 = 2.1$, extinction of resource.

Future Work

Further bifurcation analysis of nontrivial equilibria will be performed for Equation (2), and the parameter ranges for existence of multiple equilibria will be identified. The third bifurcation parameter, $\tilde{\nu}_1$, will be studied further. Ultimately we will have a better and hopefully complete understanding of the dynamics of (2).

References

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