

Math 214 Sample Homework

My Name

January 26, 2022

1. Recall the Knights and Knaves from the first day of class. Recall that knights always tell the truth, and knaves always lie. You meet three inhabitants: Patricia, Quinn and Roberta. Patricia claims that it's false that Roberta is a knave. Quinn says, 'Either Roberta is a knight or I am a knight.' Roberta says that Quinn is a knave. Who are knights and who are knaves? Prove your answer (using truth table).
2. Later, you meet Ann, Bert and Chuck. Ann says, 'Chuck could claim that I am a knight.' Bert says that only a knave would say that Ann is a knave. Chuck claims, 'Ann could say that I am a knave.' Who are knights and who are knaves? Prove your answer.
3. By using truth tables prove that, for all statements P and Q, the statement ' $P \Rightarrow Q$ ' and ' $(\text{not } Q) \Rightarrow (\text{not } P)$ ' are equivalent.

4. Prove that for all real numbers a, b and c ,

$$bc + ac + ab \leq a^2 + b^2 + c^2.$$

5. Prove that for all real numbers a and b ,

$$|a| < |b| \Rightarrow a^2 \leq b^2.$$

6. Write the set $A = \{-1, -2, -3, \dots\}$ in a form $\{x \in \mathbb{Z} : p(x)\}$.