Solve the following problems. For proof problems, please remember to use complete sentences and good grammar.

1. (4 Points) For each of the following two statements:
   (i) There exists integers $a$ and $b$ such that both $ab < 0$ and $a + b > 0$.
   (ii) For every $x \in \mathbb{R}$, there exists $y \in \mathbb{R}$, such that $xy = 1$.

Do the following:
   (a) Express the statement as a sequence of logic symbols and quantifiers.
   (b) Express the negation of the statement in both symbols and words.
   (c) Determine whether the statement is true or false.

2. (4 Points) For each of the following two quantified statements in logic symbols:
   (i) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, (y > x) \implies (y > 5)$.
   (ii) $\exists x \in \mathbb{R}, \sim ((x^2 > 0) \implies (x > 0))$.

Do the following:
   (a) Translate each statement into an English sentence.
   (b) Express the negation of the statement in both symbols and words.

3. (2 Points) Prove directly that if $x$ is an even integer, then $7x + 5$ is an odd integer.

4. (2 Points) Prove by contrapositive that if the product of two integers is even, then one of them must be even.

5. (4 Points) Let $x \in \mathbb{Z}$. Use steps or lemmas to prove that if $7x - 4$ is even, then $3x - 11$ is odd. (Hint: $x$ should be even or odd?)

6. (4 Points) Prove that if $n \in \mathbb{Z}$, then $n^3 - n$ is even. (Hint: use proof by cases)

7. (4 Points) Let $a, b \in \mathbb{N}$. Prove that if $ab$ is odd, then $a^2 + b^2$ is even.

8. (extra 2 Points) The set of complex numbers is $\mathbb{C} = \{a + bi : a, b \in \mathbb{R}\}$ where $i$ satisfies $i^2 = -1$. One can prove that $\mathbb{C}$ is a field (in the same sense as the field definition given in notes). Prove that there is no any order relation can be defined on $\mathbb{C}$ so $\mathbb{C}$ is an ordered field. (all definitions are in the notes)