Math 214 Homework 1

your name

Due date: September 8, 2017

Give the answer of the following problems. Each problem is 4 points.

1. Write each of the following sets as specified.
   (a) List the elements in the set 
   \[ A_1 = \{ n \in \mathbb{N} : n^3 < 100 \} \]
   and 
   \[ A_2 = \{ n^3 : n \in \mathbb{N}, n^3 < 100 \}. \]
   (b) Describe the set \( B = \{-3, -2, -1, 0, 1, 2, 3\} \) using the notation \( \{ n : p(n) \} \), where \( p(n) \) specifies the property of element \( n \).

2. Recall that for a set \( A \), \( \mathcal{P}(A) \) denotes the power set of \( A \).
   (a) Find \( \mathcal{P}(\mathcal{P}(\{1\})) \) and its cardinality.
   (b) Give example of a set \( S \) such that \( S \subseteq \mathcal{P}(\mathbb{N}) \) and \( |S| = 5 \).
   (c) Give example of a set \( S \) such that \( S \in \mathcal{P}(\mathbb{N}) \) and \( |S| = 5 \).

3. The following problems involve set operations.
   (a) Given an example of three sets \( A, B, \) and \( C \) such that \( B \neq C \) but \( B - A = C - A \).
   (b) Let \( A = \{ x \in \mathbb{R} : |x - 1| \leq 2 \} \), \( B = \{ x \in \mathbb{R} : |x| \geq 1 \} \) and \( C = \{ x \in \mathbb{R} : |x + 2| \leq 3 \} \). Express \( A, B \) and \( C \) using interval notation, and determine each of the following sets using interval notation: \( A \cup B, A \cap B, B - C \).

4. (a) For a real number \( r \), define \( S_r \) to be the interval \( [r - 1, r + 2] \). Let \( A = \{1, 3, 4\} \). Determine \( \bigcup_{\alpha \in A} S_{\alpha} \) and \( \bigcap_{\alpha \in A} S_{\alpha} \).
   (b) For \( n \in \mathbb{N} \), let \( A_n = \left( -\frac{1}{n}, 2 - \frac{1}{n} \right) \). Determine \( \bigcup_{n \in \mathbb{N}} A_n \) and \( \bigcap_{n \in \mathbb{N}} A_n \).

5. For two sets \( A \) and \( B \), recall that \( A \times B \) is the Cartesian product of \( A \) and \( B \).
   (a) Let \( A = \{a, b\} \). Determine and list all elements of \( A \times \mathcal{P}(A) \).
   (b) Let \( A = \{0, 1\} \) and \( B = [0, 2] \cap [1, 3] \). Describe the graph of \( A \times B \) (such as a circle or a line etc.)
   (c) Let \( A = \{0, 1\}, B = (0, 1) \cap A \) and \( C = \mathbb{R} \). What is \( A \times B \times C \)?

6. Determine and list all different partitions of the set \( \{1, 2, 3, 4\} \).

7. (extra 2 points) Determine the cardinality of the set of all different partitions of the set \( S_n = \{ k \in \mathbb{N} : 1 \leq k \leq n \} \), where \( n \) is any positive integer. For example, \( |S_1| = 1, |S_2| = 2, \) and \( |S_4| \) is solved in the last problem. If an explicit formula of \( |S_n| \) cannot be obtained, find an implicit expression of \( |S_n| \).