Z. Angew. Math. Phys. 64 (2013), 1279–1281

0044-2275/13/041279-3
published online May 15, 2013
DOI 10.1007/s00033-013-0336-y

Zeitschrift für angewandte Mathematik und Physik ZAMP

ERRATUM

Erratum to: Existence and uniqueness of steady state solutions of a nonlocal diffusive logistic equation

Linan Sun, Junping Shi and Yuwen Wang

Erratum to: Z. Angew. Math. Phys. DOI 10.1007/s00033-012-0286-9

We show that the existence of a principal eigenvalue of a linear differential operator claimed in [4] does not always hold; hence, the proof of the stability and uniqueness of positive steady-state solution in [4] are not correct.

For the linearized operator ($\phi \in X = \{v \in C^2[-1,1] : v(\pm 1) = 0\}$)

$$L[\phi] = \phi''(x) + \lambda\phi(x) - \lambda\phi(x)\int_{-1}^{1} f(x,y)u(y)\mathrm{d}y - \lambda u(x)\int_{-1}^{1} f(x,y)\phi(y)\mathrm{d}y, \tag{1}$$

where $f \in L^2((-1,1)\times(-1,1))$ is a nonnegative function, and $u \in X$ satisfying $u(x) > 0$ for $x \in (-1,1)$, the following property was stated in [4]: The principal eigenvalue μ_1 is real-valued, μ_1 is a simple eigenvalue with a positive eigenfunction $\phi_1(x)$, and eigenfunctions corresponding to other eigenvalues are sign-changing. This is not always correct. A counterexample is as follows: Let

$$f(x,y) \equiv 1, \quad \text{and} \quad u(x) = \frac{(4\lambda - \pi^2)\pi}{16\lambda}\cos\frac{\pi x}{2},$$

then from Section 2 of [4], $u(x)$ is the positive steady-state solution of (1.5) in [4] for $\lambda > \pi^2/4$. It is straightforward to calculate all the eigenvalues and associated eigenvalues of L in this case:

$$\begin{aligned}
\mu_1 &= \frac{\pi^2}{4} - \lambda, \;\; \phi_1(x) = \cos\frac{\pi x}{2}, \\
\mu_{2j} &= \frac{\pi^2}{4}(1 - 4j^2), \;\; \phi_{2j}(x) = \cos(j\pi x), \;\; j \geq 1, \\
\mu_{2j+1} &= -(j^2 + j)\pi^2, \;\; \phi_{2j+1}(x) = \cos\frac{\pi x}{2} + c_{2j+1}\cos\frac{(2j+1)\pi x}{2}, \;\; j \geq 1,
\end{aligned}$$

where c_{2j+1} satisfies

$$\left(\lambda - \frac{\pi^2}{4}\right)\left(\frac{c_{2j+1}}{2j+1} + 1\right) - \pi^2(j^2 + j) = 0.$$

The online version of the original article can be found under doi:10.1007/s00033-012-0286-9.

Birkhäuser

This indeed is a special rescaled case of Lemma 3.1 in [2]. In particular, $\mu_1 = \frac{\pi^2}{4} - \lambda < -\frac{3\pi^2}{4} = \mu_2$ if $\lambda > \pi^2$. Hence, μ_1 (the only eigenvalue with a positive eigenfunction) is not the largest eigenvalue of L if $\lambda > \pi^2$, and in this case, L does not have a principal eigenvalue. Another example of nonlocal problem which does not possess a principal eigenvalue can be found from [1]. We remark that if $\lambda u(x) < 0$ and f is nonnegative, then a principal eigenvalue μ_1 of L exists from Theorem 2.2 of [3]. More generally, the eigenvalue problem for a bounded domain Ω in $\mathbb{R}^n$

$$\begin{cases} \Delta\phi + m(x)\phi + a(x)\int_\Omega b(x,y)u(y)\mathrm{d}y = -\mu\phi, & x \in \Omega, \\ \phi = 0, & x \in \partial\Omega, \end{cases} \tag{2}$$

possesses a principal eigenvalue if a, b are nonnegative (Theorem 2.2 of [3]), but not, in general, when $a \le 0$ and $b \ge 0$.

As a consequence, the stability results in Theorem 3.1 and the uniqueness result in Corollary 3.2 of [4] no longer hold. Theorem 3.3 is now stated as

Theorem 3.3. *Suppose that f satisfies (f1) and (f2). Then*

1. *Any positive solution $u(\lambda, x)$ of (2.7) is an even function for $x \in (-1, 1)$, and $u(\lambda, x)$ is strictly decreasing in x for $x \in (0, 1]$.*
2. *For every $\lambda > \pi^2/4$, (2.7) has a positive solution $u(\lambda, x)$, and it satisfies*

$$f_1(0)\int_{-1}^{1} u(x)\mathrm{d}x \le 1 - \frac{\pi^2}{4\lambda}.$$

In the Remark 3.4 of [4], we claimed that the symmetry of the solution follows from the uniqueness (which is not known now). But indeed, the symmetry of the solution can follow from the moving plane method used in the proof of Theorem 3.3; hence, it is still valid without knowing the uniqueness.

We comment that when a principal eigenvalue of L does exist, then the results in Theorem 3.1 and Corollary 3.2 of [4] are proved with the proof provided there, for example, in the case of the example above when $\pi^2/4 < \lambda < \pi^2$. Indeed, the positive steady-state solution in the example above is linearly stable for all $\lambda > \pi^2/4$ as all eigenvalues are explicitly calculated and they are all negative. However, for the general $f(x, y)$, the stability and uniqueness of the positive steady state remain open now.

Acknowledgments

We thank Professor Antonio Suárez's communication about this problem and also an anonymous referee for helpful comments and suggestions.

References

1. Allegretto, W., Barabanova, A.: Positivity of solutions of elliptic equations with nonlocal terms. Proc. R. Soc. Edinburgh Sect. A **126**(3), 643–663 (1996)
2. Chen, S.S., Shi, J.P.: Stability and Hopf bifurcation in a diffusive logistic population model with nonlocal delay effect. J. Differ. Equ. **253**(12), 3440–3470 (2012)
3. Corrêa, F.J.S.A., Delgado, M., Suárez, A.: Some non-local problems with nonlinear diffusion. Math. Comput. Model. **54**(9–10), 2293–2305 (2011)
4. Sun, L.N., Shi, J.P., Wang, Y.W.: Existence and uniqueness of steady state solutions of a nonlocal diffusive logistic equation. Zeitschrift fuer Angewandte Mathematik und Physik (ZAMP), doi:10.1007/s00033-012-0286-9

Linan Sun, Junping Shi and Yuwen Wang
Y.Y.Tseng Functional Analysis Research Center
Harbin Normal University
Harbin, 150025, Heilongjiang
People's Republic of China
e-mail: sunlinan666666@yahoo.com.cn

Yuwen Wang
e-mail: wangyuwen2003@sohu.com

Linan Sun
Department of Mathematics
Heihe University
Heihe, 164300, Heilongjiang
People's Republic of China

Junping Shi
Department of Mathematics
College of William and Mary
Williamsburg, VA, 23187-8795, USA
e-mail: shij@math.wm.edu