# A Model for Blue Crab Population in the Chesapeake Bay

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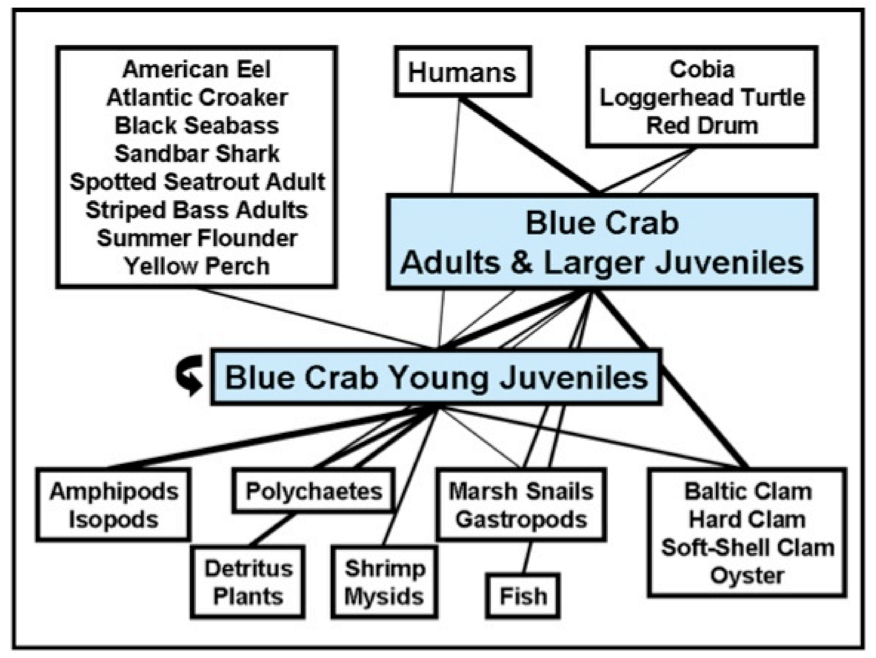
## Abstract

We model the population of the Blue Crab in the Chesapeake Bay by using differential equations. Blue crabs are inherently cannibalistic of juveniles, while also in competition with juvenile blue crabs for resources. These differential equations describe the intraguild predation consistent in the blue crab food web, as well as the cannibalistic nature of the blue crab. We introduce an aging and birth rate to alter an intraguild predation model to fit the cannibalistic nature.

## Blue Crab–Intraguild Predation

The blue crab partakes in intraguild predation, which is a subset of omnivory. Omnivory is commonly defined as predation over more than one trophic level[1]. In this case, we have the blue crab, which eats both juvenile blue crabs as well as the bivalves and clams that juvenile blue crabs eat[2](pg. 592). In accordance with intraguild predation, the blue crab is inherently cannibalistic[2](pg. 620). Thus, in terms of intraguild predation, the predator is the blue crab adult, while the prey is the blue crab juvenile.




## Blue Crab Diet

The diet for the blue crab is important to this study so as to accurately determine the population dynamics of the resource, for which the juvenile and adult blue crab will compete. Adult blue crabs consume over 99 species, mainly mollusks ($20-40\%$ of stomach content), arthropods ($10-26\%$), chordates ($5-12\%$), and annelids ($1-7\%$), along with juvenile blue crabs[2](pg. 592). Juvenile blue crabs mainly feed on bivalves, crustaceans, and detritus, or plant matter[2](pg. 548). From this, we can say for this study that the resource could be considered to be from the mollusk phylum, with a specific point of emphasis on bivalves.

## Model Analysis

The Blue Crab is a cannibalistic predator, but one that competes with its offspring for food as well. Accordingly, the model for the Blue Crab must accurately represent the intraguild predation that is present, as well as the cannibalism inherent in the Blue Crab population. We have the following model of the cannibalistic intraguild predation modified from the one of Verdy and Amarasekere [3] as such:

$$\begin{aligned}\frac{dR}{dt} &= rR\left(1-\frac{R}{K}\right)-\frac{aRN}{ahR+1}-\frac{a'RP}{a'h'R+\alpha\eta N+1},\\ \frac{dN}{dt} &= \frac{b'a'RP}{a'h'R+\alpha\eta N+1}-\frac{\alpha NP}{a'h'R+\alpha\eta N+1}-mN-eN,\\ \frac{dP}{dt} &= eN-\frac{m'}{a'h'R+\alpha\eta N+1}P,\end{aligned} \qquad (1)$$

where $R$ is the resource, $N$ is the prey, and $P$ is the predator. The system parameters are defined in the following table:

| *Parameter* | *Meaning* |
|---|---|
| $r$ | Resource Growth Rate |
| $K$ | Carrying Capacity |
| $a$ | Rate of Consumption of the Resource by the Juveniles |
| $h$ | Handling Time of the Juveniles for Resource Consumption |
| $a'$ | Rate of Consumption of the Resource by the Adults |
| $h'$ | Handling Time of the Adults for Resource Consumption |
| $\alpha$ | Rate of Consumption of the Juveniles by the Adults |
| $\eta$ | Handling Time of the Adults for Juvenile Consumption |
| $m$ | Intrinsic Mortality Rate of the Juveniles |
| $e$ | Aging Rate of the Juveniles |
| $b'$ | Rate of Efficiency of Resource Consumption by Adults |
| $m'$ | Intrinsic Mortality Rate of the Adults |

Table : Parameters

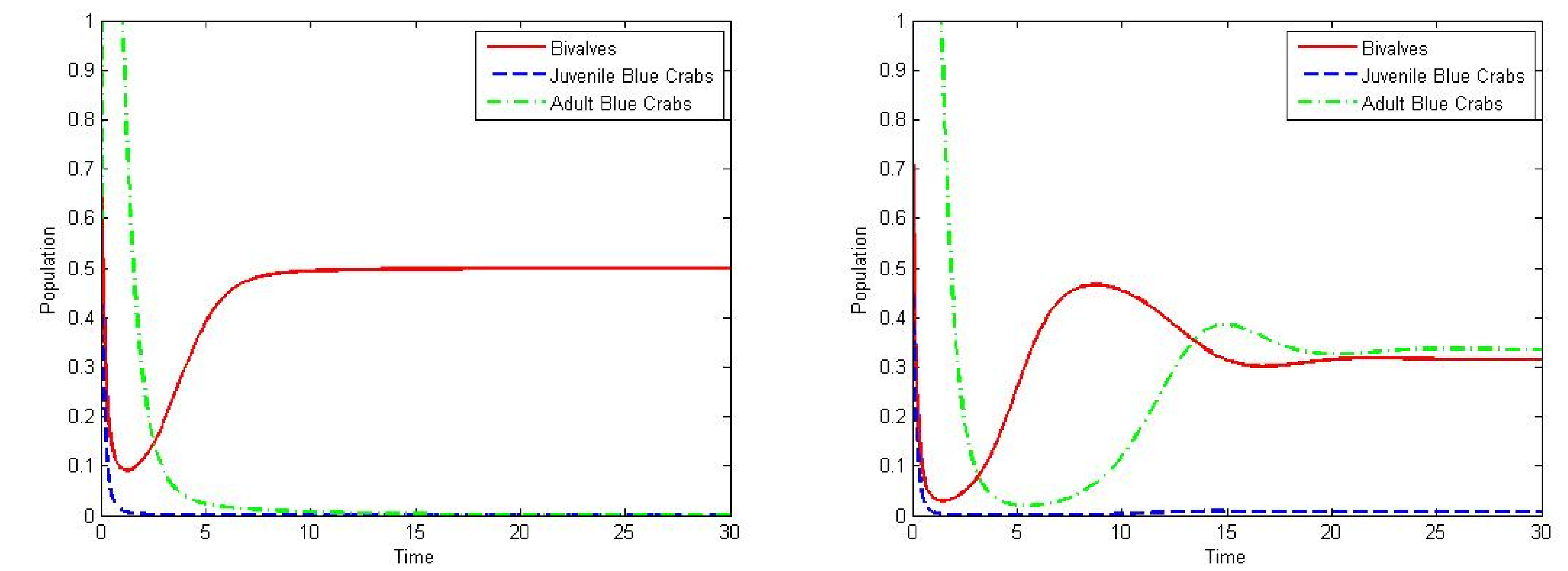
After non-dimensionalization, we have the new dimensionless system:

$$\begin{aligned}\frac{dU}{ds} &= U\left(1-\frac{U}{k}\right)-\frac{cUV}{U+1}-\frac{\lambda UW}{\theta U+V+1},\\ \frac{dV}{ds} &= -\frac{VW}{\theta U+V+1}-M_1V-EV+\frac{\delta UW}{\theta U+V+1},\\ \frac{dW}{ds} &= EqV-\frac{M_2W}{\theta U+V+1},\end{aligned} \qquad (2)$$

## Equilibrium and Stability Analysis

System (2) has two non-negative equilibria with at least one component being zero: $(0,0,0)$ and $(k,0,0)$. The trivial equilibrium, $(0,0,0)$ is a saddle point, and, as such, is unstable. Using the Routh-Hurwitz Stability Criterion, and using $q$ as a bifurcation parameter, we see that the semi-trivial equilibrium $(k,0,0)$ is locally stable when $q<\frac{(M_1+E)M_2}{E\delta k}\equiv q^*$, and is unstable when $q>q^*$. The positive equilibria can be solved from (2). Indeed the equilibrium equations can be reduced to a quadratic equation of the variable $U$. Hence (2) has at most 2 positive equilibria.

## Numerical Simulations




Parameter used: $k=0.5$, $c=20$, $\lambda=1$, $\theta=1$, $M_1=3$, $E=2$, $\delta=0.5$, $M_2=1.8$.
Initial value: $(U,V,W)=(0.8,0.5,0.2)$. Here the threshold value $q^*=18$.
(Left Panel): $q=15$, extinction of blue crab;
(Right Panel): $q=30$, persistence of blue crab.

## Future Work

Further bifurcation analysis of nontrivial equilibria will be performed for Equation (2), and the parameter ranges for existence of multiple equilibria will be identified. Ultimately we will have a better and hopefully complete understanding of the dynamics of (2). Moreover we will also consider a larger 4-variable system which also incorporates the effect of the top level predator such as cobia, or even human beings[2](pg. 620).

## References

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[3] Verdy, A.; Amarasekare, P. Alternative stable states in communities with intraguild predation. *Jour. Theo. Biol.* **262** (2010), 116–128.

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