Assignment 4

Math 345, Prof. Shi

Due: Friday, Oct 5 (5pm)

1. Consider the first order ODE: $\dfrac{dy}{dt} = y(y-1)^2$.

   (a) Find the steady state solutions, and determine the stability (stable, unstable, or cannot be linearly determined).

   (b) Sketch the direction field of the system and sketch the solution curves in (t, y) plane. (see Page 168)

   (c) Sketch the graph of $f(y) = y(y-1)^2$ versus y, and determine when the solution $y(t)$ is increasing or decreasing. (see Page 169)

   (d) Sketch the phase line. (see Page 169)

   (e) Use `Matlab` program `Dfield` to plot the direction field for the display window $-1 \le t \le 5$, $-1 \le y \le 3$.

2. Page 155 Problem 16 (c) and (d) with more questions.

   (a) Find the steady states of the following systems of equations.

   (b) Determine the Jacobian of the system for each steady states, and solve the trace, determinant, and eigenvalues of Jacobian.

   (c) Determine the types of each steady states by using the trace and determinant of the Jacobian matrix (stable node, unstable node, stable spiral, unstable spiral, saddle, or cannot be linearly determined etc.)

   (d) Find the nullclines of the systems, and sketch the nullclines. Indicate the direction of vector field on the nullcline and in the regions bounded by the nullclines.

   (e) Use `Matlab` program `Pplane` to plot the phase portraits for the display window $-1 \le x \le 3$, $-1 \le y \le 3$.

$$(1) \quad \begin{aligned} \frac{dx}{dt} &= x - x^2 - xy, \\ \frac{dy}{dt} &= y(1-y). \end{aligned} \qquad (2) \quad \begin{aligned} \frac{dx}{dt} &= x - xy, \\ \frac{dy}{dt} &= xy - y. \end{aligned}$$

3. Page 155 Problem 14.

   (a) You do not explain this part, but you will use the equation in following parts.

   (b) Use a table as in problem 4 in Homework 3 to list the dimensions of all parameters and variables.

   (c) Use change of variables: $t^* = \mu t$, $N^* = \dfrac{\alpha}{C_0} N$, and $C^* = \dfrac{1}{k_n} C$ to obtain a new equation with dimensionless equations of N^* and C^*.

   (d) Find all steady states of the dimensionless equation from (c).

   (e) We will not do this part in this assignment.

4. For each linear system,

(a) Find the general **real-valued** solution.

(b) Determine the types of steady state $(0, 0)$ (stable node, unstable node, stable spiral, unstable spiral, saddle, or neutral center etc.)

(c) Sketch by hand the behavior of solutions on (x, y) phase plane.

(d) Use `Matlab` program `Pplane` to plot the phase portraits for the display window $-2 \le x \le 2$, $-2 \le y \le 2$.

(e) Use `Matlab` program `Pplane` to plot the time series of solutions with initial condition $(x(0), y(0)) = (1, 1)$.

$$(1) \quad \begin{aligned} \frac{dx}{dt} &= -x + 4y, \\ \frac{dy}{dt} &= -2x + 5y. \end{aligned} \qquad (2) \quad \begin{aligned} \frac{dx}{dt} &= 2x + 2y, \\ \frac{dy}{dt} &= -4x + 6y. \end{aligned}$$