Assignment 3

Math 345, Prof. Shi

Due: Friday, Sept 28 (5pm)

1. Consider the Ricker Fisheries Model:

$$x_{n+1} = x_n e^{r(1-x_n)}.$$

   Here x_n is the number of fishes in year n, $r > 0$ is the production rate.

   (a) Find all non-negative equilibria of the model.

   (b) Determine the stability of each equilibria according to the value of r.

   (c) What is the bifurcation point of r where the stability of either equilibrium changes? What kind of bifurcation occur there? Use such information to sketch a bifurcation diagram with r as bifurcation parameter for the model.

   (d) Use Matlab to simulate the time series of solutions with $r = 1$, $r = 2.2$ and $r = 3$, and describe the behavior of solutions.

   (e) Use Matlab to plot a bifurcation diagram for $0 \le r \le 4$. At what value of r, the dynamics becomes chaotic?

2. Consider a Beverton-Holt model with harvesting:

$$x_{n+1} = \frac{2x_n}{1 + x_n} - h.$$

   Here $h \ge 0$ is the harvesting rate. Find the equilibria of the model and find the bifurcation point $h = h_0$. identify the type of bifurcation.

3. A 1960 issue of Science magazine included an article by von Foerster and his colleagues P. M. Mora and L. W. Amiot proposing a formula representing a best fit to available historical data on world population; the authors then predicted future population growth on the basis of this formula. The formula gave 2.7 billion as the 1960 world population and predicted that population growth would become infinite by Friday, November 13, 2026 - von Foerster's 115th birthday anniversary - a prediction that earned it the name "the Doom's Day Equation."

   (a) The doom's day equation which von Foerster et.al. proposed is $\frac{dN}{dt} = kN^2$ where $N(t)$ is the population at time t and $k > 0$. Solve the equation to express N as a function of t and $N(0) = N_0$.

   (b) Find the time t_* when the solution tends to infinity. Assuming that $t = 0$ is the year 1960 and $N_0 = 2.7$ billion, and $t_* = 66$ (the year 2026), find the value of k. What is the dimension and unit of k?

   (c) Use the value of k solved in part (b) and $N_0 = 2.7$ billion to find the population today (year 2018). How does this estimate compare to the real population today? (Google to find out the population today)

4. Consider a population model: $\frac{dx}{dt} = kx\left(1 - \frac{x}{N}\right) - M,\ x(0) = x_0$, where k, M, N, x_0 are positive parameters.

   (a) In the following table, fill in the dimensions of all parameters in terms of the dimensions of variables.

Variable	Dimension	Parameter	Dimension
t	τ	k	
x	λ	M	
		N	
		x_0	

   (b) Use the change of variable:

$$y = \frac{x}{N}, \quad s = kt.$$

   Derive the new equation (including the initial condition) in the new variables y and s.

5. Suppose that $N(t)$ denotes the size of a population at time t. The population evolves according to the logistic equation but, in addition, predation reduces the size of the population so that the rate of change is given by $\frac{dN}{dt} = N\left(1 - \frac{N}{50}\right) - \frac{9N}{5+N}$.

   (a) Find the equilibrium points of the equation.

   (b) Determine the stability of the equilibrium points by using linearization.

   (c) Sketch the phase line. If $N(0) = 35$, what is the limit of $N(t)$ as $t \to \infty$?