Test 2 practice

Math 212

1. Suppose that a bug is crawling on a flat plate along the circle $x = \sin t$, $y = \cos t$, while the temperature of the plate is given by $T = x^2 e^y - 2xy$. Find dT/dt at $t = \pi/4$ by the chain rule.

2. Consider function
$$f(x, y) = 48xy - 32x^3 - 24y^2$$
   (a) At $P(1, 1)$, what is the gradient of f?
   (b) Find the directional derivative of f at P along $\mathbf{A} = \mathbf{i} + 2\mathbf{j}$.
   (c) Along which direction, the value of function $f(x, y)$ decreases the fastest at P?
   (d) What is the level curve of $f(x, y)$ at P? Find the equation of the tangent line to the level curve of $f(x, y)$ at P.

3. Find an equation for the tangent plane to the surface defined by $xy^2 + 2z^2 = 12$ at the point $(1, 2, 2)$.

4. Consider function
$$f(x, y) = 4xy - x^2 y - xy^2$$
   (a) Find all the local maxima, local maxima and saddle points of the function.
   (b) Find the absolute maxima and minima of $f(x, y)$ on the closed triangular region bounded by the x-axis, y-axis and $x + y = 6$.

5. Use the method of Lagrange multipliers to find extreme values of $f(x, y) = 5xy$ on the ellipse $4x^2 + y^2 = 4$.

6. The temperature distribution on the surface $x^2 + y^2 + z^2 = 1$ is given by $T(x, y, z) = xz + yz$. Find the hottest spot and coldest spot on the surface (maximum and minimum of $T(x, y, z)$).

7. Consider the integral
$$\int_0^{\pi} \int_0^{x} x \sin y dy dx.$$
   Over which region is this integral calculated?
   (A) A rectangle with vertices $(0, 0)$, $(0, \pi)$, $(\pi, 0)$ and (π, π);
   (B) A triangle with vertices $(0, 0)$, $(0, \pi)$ and $(\pi, 0)$;
   (C) A triangle with vertices $(0, 0)$, $(0, \pi)$ and (π, π);
   (D) A triangle with vertices $(0, 0)$, $(\pi, 0)$ and (π, π).

8. Still consider the integral
$$\int_0^{\pi} \int_0^{x} x \sin y dy dx.$$
   If we first integrate with respect to x, then which is the correct form?

(A) $\int_0^{\pi} \int_0^{y} x \sin y dx dy$, (B) $\int_{-\pi}^{0} \int_0^{y} x \sin y dx dy$, (C) $\int_0^{\pi} \int_y^{\pi} x \sin y dx dy$, (D) $\int_{-\pi}^{0} \int_0^{y} x \sin y dx dy$

9. Find the volume of the region in the first octant between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0$, $x = 1$, $y = 0$ and $y = 1$.

10. Calculate the integral:

$$\iint_D \frac{y}{1+x^2} dA$$

where D is the bounded by $y = \sqrt{x}$, $y = 0$ and $x = 1$.

11. Calculate the iterated integral by first reversing the order of integration.

$$\int_0^1 \int_x^1 \cos(y^2) dy dx$$

12. Find the volume of the tetrahedron in the first octant bounded by the three coordinate planes, and the plane $3x + 4y + 5z = 60$.

13. Find the volume under the paraboloid $z = x^2 + y^2$ above the triangle enclosed by the lines $y = x$, $x = 0$ and $x + y = 2$ in the xy plane.