Exam 2 make up problems

Math 212

Rule: This set of problems is similar to the problems in Test 1; The purpose of doing the problems is to solidify your knowledge and problem-solving ability for these material. It is voluntary to do the problem and hand in, but it is strongly suggested for the students whose Test 1 score is below 80; You need to complete the problems by yourself, but you can consult books and notes, and there is no time limit; Handed in problems will be graded by Prof. Shi; If your score in Test 1 is n, you can receive up to $(100 - n) \times 0.3$ extra points for the problems. (For example, if your score is 70, and you do all the problems here correctly, then you can get 9 extra points, and your modified test score is 79.)

1. (16 pt) Consider the function $f(x, y) = x^2y^2 - x$.
   (a) What is the directional derivative of f at $(2, 1)$ in the direction of the vector $\mathbf{v} = \langle -1, 1\rangle$?
   (b) What is the maximum directional derivative of f at $(2, 1)$, and in which direction does it occur?
   (c) Write the equation for the tangent plane to the graph of $z = f(x, y)$ at $(2, 1, 2)$.
2. (20 pt) Consider the function $f(x, y) = x^4 + y^4 - 4xy + 1$.
   (a) Find all critical points of $f(x, y)$ and determine their type *i.e.* local min, local max or saddle point.
   (b) Find the maximum and minimum values of $f(x, y)$ in the rectangle R with vertices $(2, 2)$, $(-2, 2)$, $(-2, -2)$ and $(2, -2)$.
3. (16 pt) Find the points on the surface $2x^2 + y^2 = (z - 2)^2$ that are (a) closest to the origin and (b) furthest from the origin.
4. (16 pt) Use the Lagrange multipliers to find the maximum and minimum values the absolute maximum and minimum values of $f(x, y, z) = xyz$ subject to the constraint $x^2+2y^2+3z^2 = 6$.
5. (16 pt) Consider the iterated integral $\displaystyle\int_0^3 \int_{x^2}^9 xe^{-y^2} dydx$.
   (a) Sketch the region of integration.
   (b) Reverse the order of integration properly.
   (c) Evaluate the integral from part (b).
6. (16 pt) Calculate the integral:
$$\iint_D x^2 dA$$
   where D is the region bounded by $y = x^2 - 2x$ and $y = -x^2 + 4$.