Test 1 practice

Math 212

1. Which statement is correct? (choose only one answer.)
   (A) $|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}| \cdot |\mathbf{B}| \cdot \cos\theta$; (B) $|\mathbf{A} \cdot \mathbf{B}| \leq |\mathbf{A}| \cdot |\mathbf{B}|$;
   (C) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = -(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$; (D) $\mathbf{i} + \mathbf{j}$ is parallel to $\mathbf{i} - \mathbf{j}$.

2. The plane of through $(-3, 4, 1)$ and lying in a plane parallel to xy-plane is
   (A) $z = 1$; (B) $y = 4$; (C) $x = -3$; (D) $x = -3$, $y = 4$; (E) $x = -3$, $z = 1$;
   (F) $y = 4$, $z = 1$; (G) None of the above.

3. Let $\mathbf{v}(t)$ be a differentiable function. If $|\mathbf{v}(t)| = 3$ for all t, then which is always true?
   (A) $\mathbf{v}'(t) = \mathbf{0}$ for all t; (B) the path of $\mathbf{v}(t)$ must be a circle;
   (C) $\mathbf{v}(t) \cdot \mathbf{v}'(t) = 0$ for all t; (D) the path of $\mathbf{v}(t)$ must be a line;

4. The domain of
   $$f(x, y) = \sqrt{x} - \frac{1}{y}$$
   is
   (A) $x > 0$, $y > 0$, (B) $x \geq 0$, $y > 0$, (C) $x \geq 0$, $y \neq 0$, (D) $x > 0$, $y \neq 0$.

5. Let $\mathbf{A} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{B} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$.
   (a) Find the area of the parallelogram determined by $\mathbf{A}$ and $\mathbf{B}$.
   (b) Find a vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.
   (c) Find the projection of $\mathbf{A}$ along $\mathbf{B}$.

6. Find the equation of the plane passing through the point $(0, 1, -2)$ and containing the line $\mathbf{r}(t) = \langle 0, 2, -3\rangle + t\langle 1, 0, -1\rangle$.

7. Suppose that the plane M contains points $P(3, 0, 4)$, $Q(2, -3, 1)$ and $R(0, 0, 1)$, and the line L contains points P and R.
   (a) Find the equation of plane M.
   (b) Find the equation of line L.
   (c) Find the angle $\angle PQR$.
   (d) Find the equation of the line where M and plane $2x - y - z = 4$ intersect.
   (e) Find the area of the triangle ΔPQR.

8. The motion of a particle is $\mathbf{r}(t) = (2 + t)\mathbf{i} + (t - t^2)\mathbf{j} + (t^2 - 3)\mathbf{k}$.
   (a) Find the velocity $\mathbf{v}(t)$ and the speed $s(t)$ of the motion;
   (b) Find the acceleration $\mathbf{a}(t)$ of the motion;
   (c) Find the equation of the tangent line of $\mathbf{r}(t)$ at $t = 1$.

9. Find the length of the curve $\mathbf{r}(t) = t\mathbf{i} + (2/3)t^{3/2}\mathbf{k}$, $0 \leq t \leq 8$.

10. Evaluate the limit or show that it does not exist.

$$\lim_{(x,y)\to(2,2),x+y\neq 4} \frac{x+y-4}{\sqrt{x+y}-2}, \quad \lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+2y^2}$$

11. Find all first and second order partial derivatives of $f(x,y) = 4x^3 - xy^2$.

12. (a) Find the partial derivatives z_x and z_y if $z + x^2 - 4y^2 - 1 = 0$

    (b) Find the equation of the tangent plane to the surface $z + x^2 - 4y^2 - 1 = 0$ at the point $(2, 1, 1)$.

13. Let $y = uv$, where u and v are positive independent variables. If u measured with an error of 2% and v with an error of 3%, about what is the percentage error in the calculated value of y?

14. A base ball is thrown toward a player with an initial speed of 20 m/s at an angle of 45° with the horizontal. At the moment the ball is thrown, the player is 50 m from the thrower. At what speed and in what direction must he run to catch the ball at the same height at which it was released?