# Test 1 make up problems

Math 212

Rule: This set of problems is similar to the problems in Test 1; The purpose of doing the problems is to solidify your knowledge and problem-solving ability for these material. It is voluntary to do the problem and hand in, but it is strongly suggested for the students whose Test 1 score is below 80; You need to complete the problems by yourself, but you can consult books and notes, and there is no time limit; Handed in problems will be graded by Prof. Shi; If your score in Test 1 is $n$, you can receive up to $(100 - n) \times 0.3$ extra points for the problems. (For example, if your score is 70, and you do all the problems here correctly, then you can get 9 extra points, and your modified test score is 79.)

1. (a) Find a unit vector that is orthogonal to both $\mathbf{u} = \mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$.
   (b) Find the vector projection of $\mathbf{v}$ onto $\mathbf{u}$.

2. Let $P = (3, -1, 1)$, $Q = (4, 0, 2)$ and $R = (6, 3, 1)$.
   (a) Find the Cosine of angle $\angle PQR$.
   (b) Find the area of parallelogram generated by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$

3. (a) Find the equation of the plane through the point $(6, 0, -2)$ and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$.
   (b) Find the distance from $(0, 0, 0)$ to the plane $P$ in part (a).

4. Let $C$ be the curve with equations $x = 2 - t^3$, $y = 2t - 1$, $z = \ln t$.
   (a) Find the point where $C$ intersects the $xz$-plane;
   (b) Find the equation of the tangent line at $(1, 1, 0)$.

5. The motion of a projectile on a small planet has acceleration $\mathbf{a} = \mathbf{i} - 2\mathbf{j}$, and the projectile is fired from ground with an initial velocity $\mathbf{v}_0 = 5\mathbf{i} + 12\mathbf{j}$.
   (a) (6 pt) Find the velocity vector $\mathbf{v}(t)$ and the position vector $\mathbf{r}(t)$ of the projectile assuming its initial position is $\mathbf{r}_0 = 0\mathbf{i} + 0\mathbf{j}$.
   (b) (6 pt) When will the projectile reach the highest point, and what is the height of the projectile at that time?
   (c) (6 pt) When will the projectile hit the ground again, and how far has the projectile traveled horizontally by that time?

6. Determine whether the limit exists or not, and if it exists, find the limit: $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2 + y^4}$

7. (a) Find $f_x$, $f_y$, $f_{xx}$, $f_{xy}$ and $f_{yy}$ if $f(x, y) = \ln(3x + y^2)$.
   (b) Write the equation for the tangent plane to the graph of $f$ at $(x, y, z) = (1, 1, \ln 4)$.

8. A rectangular box has a height of 4 feet to within an accuracy of 1 in and a square base with width 2 feet to within an accuracy of 1 in. Find the volume $V$ and the approximate error $dV$ in the volume of the can.