

Multivariable Calculus Readiness Test

Math 212, Prof. Shi

Name:

1. Find the following limits using limit laws and theorems if they exist.

(a) $\lim_{x \rightarrow 0} \frac{\sqrt{2-x} - \sqrt{2}}{2x}$; (b) $\lim_{x \rightarrow 0} \frac{\sin(2x) \tan(x)}{x^2}$

2. Find the derivative of $f(x)$ by using differentiation rules and basic derivative formulas.

(a) $f(x) = \cos(\ln x)$; (b) $f(x) = \sqrt{5-x} \tan x$; (c) $f(x) = \frac{x^2 - 3}{5^x}$.

3. Consider the equation $3x^3 + y^3 - xy^2 = 7$.

(a) Find $\frac{dy}{dx}$ by implicit differentiation.

(b) Find the tangent line to the curve at $(1, 2)$.

4. Let $f(x) = \frac{x^2 + x + 2}{x - 1}$.

(a) Find all critical numbers of f .

(b) Find the intervals on which f is increasing or decreasing.

(c) Find the x -coordinates of any local maximum and local minimum points of f .

(d) Find the x -coordinates of any inflection points.

(e) Find the intervals on which f is concave upward or concave downward.

(f) Find the absolute maximum and absolute minimum values of f on $[2, 4]$.

5. Evaluate each integral.

(a) $\int_1^3 (1 + 2x - 4x^3) dx$; (b) $\int_1^4 \sqrt{x} dx$; (c) $\int \frac{\sin \sqrt{t}}{\sqrt{t}} dt$; (d) $\int \frac{1+x}{1+x^2} dx$;

(e) $\int \frac{1+x^2}{1+x} dx$; (f) $\int \sqrt{x} \ln x dx$; (g) $\int \cos^3(x) \sin(x) dx$; (h) $\int_{-5}^5 \sqrt{25-x^2} dx$;

(i) $\int \frac{3x-5}{x^2-5x+6} dx$; (j) $\int_1^\infty \frac{1}{x^2} dx$