Ex1 (1) $f(x,y) = x^5 + 3x^3y^2 + 3xy^4$

$f_x = 5x^4 + 9x^2y^2 + 3y^4 \qquad f_y = 6x^3y + 12xy^3$

$f_x(1,2) = 5\cdot 1^4 + 9\cdot 1^2\cdot 2^2 + 3\cdot 2^4 = 5 + 36 + 48 = 89$

(2) $f(x,y) = x^y$

$$\left( \begin{array}{l} (x^n)' = nx^{n-1} \\ (a^x)' = a^x\cdot \ln a \end{array} \right)$$

$f_x = yx^{y-1} \qquad f_y = x^y\cdot \ln x$

$f_x(1,2) = 2\cdot 1^{2-1} = 2$

(3) $f(x,y,z) = e^{xy}\ln z$

$f_x = ye^{xy}\ln z, \quad f_y = ye^{xy}\ln z, \quad f_z = e^{xy}\cdot \frac{1}{z}.$

$f_x(1,1,1) = 1\cdot e^1\cdot \ln 1 = 0.$

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(a) $\frac{\partial h}{\partial v}$ = the rate of change of height w.r.t. wind speed

$\frac{\partial h}{\partial t}$ = ——— w.r.t time

(b) $f_v(40,15) \approx \frac{f(40,15) - f(30,15)}{10} = \frac{25-16}{10} \approx 0.9$

$f_t(40,15) \approx \frac{f(40,15) - f(40,10)}{5} = \frac{25-21}{5} \approx 0.8$

(c) 0 since there is no change for large $t$.

**Ex2** (1) $x^2+y^2+z^2=9$

Taking derivative in $x$: $2x+0+2z\cdot\frac{\partial z}{\partial x}=0 \Rightarrow \frac{\partial z}{\partial x}=-\frac{x}{z}$

$y$: $0+2y+2z\cdot\frac{\partial z}{\partial y}=0 \Rightarrow \frac{\partial z}{\partial y}=-\frac{y}{z}$

(2) $x-z=\arctan(yz)$

Taking derivative in $x$: $1-z_x=\frac{1}{1+(yz)^2}(yz_x)$ $\quad 1=z_x\left(1+\frac{y}{1+y^2z^2}\right)$

$$\Rightarrow z_x=\frac{1}{1+\frac{y}{1+y^2z^2}}=\frac{1+y^2z^2}{1+y^2z^2+y}$$

in $y$: $0-z_y=\frac{1}{1+(yz)^2}(z+y\cdot z_y)$

$$-(1+y^2z^2)z_y=z+yz_y \Rightarrow z_y=\frac{-z}{1+y^2z^2+y}$$

**Ex3** (1) $f(x,y)=x^4y^2-2xy^5$

$f_x=4x^3y^2-2y^5 \quad f_y=2x^4y-10xy^4$

$f_{xx}=12x^2y^2 \quad f_{xy}=8x^3y-10y^4 \quad f_{yx}=8x^3y-10y^4$

$f_{yy}=2x^4-40xy^3$

(2) $z_{uvw}$ for $z=u\sqrt{v-w}$

$z_u=\sqrt{v-w} \qquad z_{uv}=\frac{1}{2\sqrt{v-w}} \qquad z_{uvw}=\frac{1}{2}\cdot\left(-\frac{1}{2}\right)(v-w)^{-\frac{3}{2}}$ $(-1)$

$$=\frac{1}{4}(v-w)^{-\frac{3}{2}}$$

Ex3 (1) $z = y\ln x$, at $(1,4,0)$

$z_x = \frac{y}{x}$ $z_y = \ln x$ $z_x(1,4) = 4$, $z_y(1,4) = 0$

$z - 0 = 4(x-1) + 0\cdot(y-4) \Rightarrow z = 4x - 4$

(2) $f(x,y) = \sqrt{x^2+y^2}$ $f_x = \frac{1}{2\sqrt{x^2+y^2}}\cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}\cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$ $f_x(3,4) = \frac{3}{\sqrt{3^2+4^2}} = \frac{3}{5}$ $f_y(3,4) = \frac{4}{\sqrt{3^2+4^2}} = \frac{4}{5}$

$L(x,y) = \cancel{f_x(3,4)}\ f(3,4) + f_x(3,4)(x-3) + f_y(3,4)(y-4)$

$= \sqrt{3^2+4^2} + \frac{3}{5}(x-3) + \frac{4}{5}(y-4) = 5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4)$

$\sqrt{(3.01)^2 + (3.98)} = f(3.01, 3.98) \approx L(3.01, 3.98)$

$= 5 + \frac{3}{5}(3.01-3) + \frac{4}{5}(3.98-4) = 5 + \frac{3}{5}\cdot 0.01 + \frac{4}{5}(-0.02) = 5 + 0.006 - 0.0016$

$= 4.999$ ~~(real value~~

(3) $dz = f_x(x_0,y_0)\underbrace{\cancel{(x-x_0)}}_{dx} + f_y(x_0,y_0)\underbrace{\cancel{(y-y_0)}}_{dy}$

$f(x,y) = x^2+y^2$ $f_x = 2x$, $f_y = 2y$ $f_x(2,5) = 4$, $f_y(2,5) = 10$

~~$dz = 4(x-2) + 10(y-5) = 4x + 10y - 58$~~

$dz = 4dx + 10dy$

$z(2.1, 5.1) \approx z(2,5) + dz = (2^2+5^2) + 4\cdot 0.1 + 10\cdot 0.1$

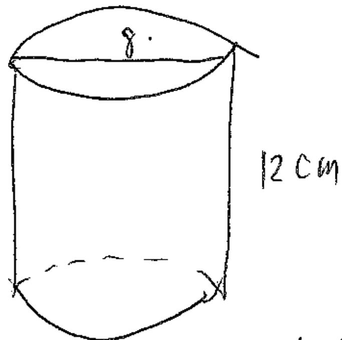
$= 29 + 1.4 = 30.4$

$dz = 1.4$

$\Delta z = z(2.1, 5.1) - z(2,5)$

$= (2.1^2 + 5.1^2) - (2^2 + 5^2) = 30.42 - 29 = 1.42$

(4)




Volume of can $= \pi r^2 h$

Volume of can with tin $= \pi (r+dr)^2 (h+dh)$

$r = 4\text{cm}, \quad h = 12\text{ cm} \qquad dr = dh = 0.04\text{ cm}$

$V(r,h) = \pi r^2 h \qquad V_r = 2\pi r h, \quad V_h = \pi r^2$

$V_r(4,12) = 2\pi \cdot 4 \cdot 12 = 96\pi, \quad V_h(4,12) = \pi \cdot 4^2 = 16\pi$

$dV = V_r(4,12)\,dr + V_h(4,12)\,dh = 96\pi\,dr + 16\pi\,dh$

So $dV = 96\pi \cdot 0.04 + 16\pi \cdot 0.04 = 112\pi \cdot 0.04 = 4.48\pi \text{ cm}^3.$