Example 1 ① $\vec{r}(t) = \langle \ln(4-t^2), \sqrt{t+1}, \cos(2t) \rangle$

$4-t^2 > 0$, $t+1 \geq 0$

$\Rightarrow 2 > t > -2$ and $t \geq -1$ $\Rightarrow$ $2 > t \geq -1$

Domain : $[-1, 2)$

② $P(-2, 4, 0)$, $Q(6, -1, 2)$

$\overrightarrow{PQ} = \langle 8, -5, 2 \rangle$

vector equation $\langle x, y, z \rangle = \langle -2, 4, 0 \rangle + t \langle 8, -5, 2 \rangle$

$0 \leq t \leq 1$

parametric equation : $x = -2 + 8t$, $y = 4 - 5t$, $z = 2t$, $0 \leq t \leq 1$

③ $\begin{cases} z = 4x^2 + y^2 \\ y = x^2 \end{cases}$ $\Rightarrow$ $z = 4x^2 + (x^2)^2 = 4x^2 + x^4$

So we can have $\vec{r}(t) = \langle t, t^2, 4t^2 + t^4 \rangle$

$-\infty < t < \infty$

④ $\vec{r}(t) = \langle \ln t, 2\sqrt{t+1}, t^2 \rangle$ at $P(0, 2\sqrt{2}, 1)$

$\downarrow$

$t = 1$

$\vec{r}\,'(t) = \langle \frac{1}{t}, \frac{1}{\sqrt{t+1}}, 2t \rangle$

$\vec{r}\,'(1) = \langle 1, \frac{1}{\sqrt{2}}, 2 \rangle$ So tangent line $\langle x, y, z \rangle = \langle 0, 2\sqrt{2}, 1 \rangle + t \langle 1, \frac{1}{\sqrt{2}}, 2 \rangle$

$\Rightarrow$ $x = t$, $y = 2\sqrt{2} + \frac{\sqrt{2}}{2} t$, $z = 1 + 2t$, $t \in \mathbb{R}$

⑤ $\vec{r}(t) = \langle \ln(4-t^2), \sqrt{t+1}, \cos(2t) \rangle$

velocity $\vec{r}\,'(t) = \langle \frac{-2t}{4-t^2}, \frac{1}{2\sqrt{t+1}}, -2\sin(2t) \rangle$

Speed $|\vec{r}\,'(t)| = \sqrt{\frac{4t^2}{(4-t^2)^2} + \frac{1}{4(t+1)} + 4\sin^2(2t)}$

acceleration $\vec{r}\,''(t) = \langle \frac{(4-t^2)(-2) - (-2t)(-2t)}{(4-t^2)^2}, \frac{-1}{4(t+1)^2}, -4\cos(4t) \rangle$

$= \langle \frac{-8-2t^2}{(4-t^2)^2}, \frac{-1}{4(t+1)^2}, -4\cos(4t) \rangle$

⑥ $\vec{a}(t) = \langle t, t^2, \cos(2t) \rangle$, $\vec{v}(0) = \langle 1, 0, 1 \rangle$, $\vec{r}(0) = \langle 0, 1, 0 \rangle$

$\vec{v}(t) = \vec{v}(0) + \int_0^t \vec{a}(s)\,ds = \langle 1, 0, 1 \rangle + \int_0^t \langle s, s^2, \cos(2s) \rangle\,ds$

$= \langle 1, 0, 1 \rangle + \langle \frac{1}{2}t^2, \frac{1}{3}t^3, \frac{1}{2}\sin(2t) \rangle$

$\vec{r}(t) = \vec{r}(0) + \int_0^t \vec{v}(s)\,ds = \langle 0, 1, 0 \rangle + \int_0^t \langle 1 + \frac{1}{2}s^2, \frac{1}{3}s^3, 1 + \frac{1}{2}\sin(2s) \rangle\,ds$

$= \langle 0, 1, 0 \rangle + \langle t + \frac{1}{6}t^3, \frac{1}{12}t^4, t - \frac{1}{4}\cos(2t) + \frac{1}{4} \rangle$

Example 2

(a) $\vec{r}(t) = \langle 2\sin t, 5t, 2\cos t \rangle \qquad -10 \le t \le 10$

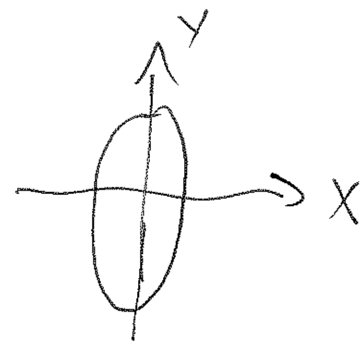
$\vec{r}\,'(t) = \langle 2\cos t, 5, -2\sin t \rangle$

Arc Length $= \int_{-10}^{10} \sqrt{(2\cos t)^2 + 5^2 + (-2\sin t)^2}\, dt = \int_{-10}^{10} \sqrt{25+4}\, dt$

$= \sqrt{29} \cdot 20 = 20\sqrt{29}$

(b) $\begin{cases} 4x^2 + y^2 = 4 \\ x + y + z = 2 \end{cases}$

$4x^2 + y^2 = 4 \Rightarrow \begin{cases} x = \cos\theta \\ y = 2\sin\theta \end{cases} \qquad 0 \le \theta \le 2\pi$



$z = 2 - x - y = 2 - \cos\theta - 2\sin\theta$

So the curve is $\begin{cases} x = \cos\theta \\ y = 2\sin\theta \\ z = 2 - \cos\theta - 2\sin\theta \end{cases} \qquad 0 \le \theta \le 2\pi$

$\vec{r}\,'(\theta) = \langle -\sin\theta, 2\cos\theta, \sin\theta + 2\cos\theta \rangle$

Arc Length $= \int_0^{2\pi} \sqrt{\sin^2\theta + 4\cos^2\theta + (\sin\theta + 2\cos\theta)^2}\, d\theta = \int_0^{2\pi} \sqrt{2\sin^2\theta + 4\sin\theta\cos\theta + 8\cos^2\theta}\, d\theta$