Math 212 Lecture 4

Example 1 line in (x, y) passing through $(1,2)$, $(-3,7)$

$$\text{slope} = \frac{7-2}{-3-1} = \frac{5}{-4}$$

Point - Slope form $\quad y-1 = -\frac{5}{4}(x-2)$ or $y = -\frac{5}{4}x + \frac{7}{2}$

parametric form $\quad \overrightarrow{PQ} = \langle -4, 5 \rangle \quad \begin{cases} x = -4t+1 \\ y = 5t+2 \end{cases}$

vector form : $\overrightarrow{PQ} = \langle -4, 5 \rangle$, Normal direction $\langle 5, 4 \rangle$

$$\langle 5, 4 \rangle \langle x-1, y-2 \rangle = 0$$

$$5(x-1) + 4(y-2) = 0$$

$$5x + 4y = 13$$

Ex 2 (1) $P(1,2,3)$, $Q(2,-4,9)$, $R(0,5,-4)$

$\overrightarrow{PQ} = \langle 1, -6, 6 \rangle \quad \overrightarrow{PR} = \langle -1, 3, -7 \rangle$

normal direction $= \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ 1 & -6 & 6 \\ -1 & 3 & -7 \end{vmatrix} = 24i + j - 3k$

plane : $\langle 24, 1, -3 \rangle \langle x-1, y-2, z-3 \rangle = 0$

$$24(x-1) + 1\cdot(y-2) + (-3)(z-3) = 0$$

$$24x - 24 + y - 2 - 3z + 9 = 0$$

$$24x + y - 3z = 17$$

② planes: $x - y + z = 4$, $3x + 4y - 5z = 0$

normal direction $\langle 1, -1, 1 \rangle$ $\langle 3, 4, -5 \rangle$

angle between planes

= angle between normal direction

$$\cos\theta = \frac{\langle 1,-1,1\rangle\langle 3,4,-5\rangle}{\sqrt{1^2+1^2+1^2}\,\sqrt{3^2+4^2+5^2}}$$

$$= \frac{3-4-5}{\sqrt{3}\cdot 5\sqrt{2}} = \frac{-6}{5\sqrt{6}} = \frac{-\sqrt{6}}{5}$$

$$\theta = \cos^{-1}\left(-\frac{\sqrt{6}}{5}\right)$$

③ point: $P(1, 2, 3)$ parallel to $2x + 4y + 7z = 18$

Two planes are parallel if their normal vectors are same

$$\langle 2, 4, 7 \rangle \cdot \langle x-1, y-2, z-3 \rangle = 0$$

$$2(x-1) + 4(y-2) + 7(z-3) = 0$$

$$2x - 2 + 4y - 8 + 7z - 21 = 0$$

$$2x + 4y + 7z = 31$$

Ex3 ① direction $\overrightarrow{PQ} = \langle -5, 5, -3 \rangle$

$P(1,2,3)$, $Q(-4,7,0)$

parametric equation: $x = -5t+1$, $y = 5t+2$, $z = -3t+3$

symmetric equation: $\frac{x-1}{-5} = \frac{y-2}{5} = \frac{z-3}{-3}$

② z- axis $\{(0,0,z) : z \in \mathbb{R}\}$

direction : $\langle 0, 0, 1 \rangle$

parametric : $x = 0,\ y = 0,\ z = t$

symmetric : $x = 0,\ y = 0,\ z.$

③ line where $x+y+z=0$ and $2x-3y+7z=9$ intersect.

Strategy 1 : normal $\vec{u}=\langle 1,1,1\rangle$ and $\vec{v}=\langle 2,-3,7\rangle$

So the direction of line $\perp$ both $\Rightarrow \vec{u}\times\vec{v}$.

Then find a point.

Strategy 2 : Solve $\begin{cases} x+y+z=0 \\ 2x-3y+7z=9 \end{cases}$ $\Rightarrow z=-x-y$

$$\Rightarrow 2x-3y+7(-x-y)=9 \Rightarrow -5x-10y=9$$

$$\Rightarrow 5x=-9-10y \Rightarrow x=-2y-\frac{9}{5}$$

$\boxed{\text{Let } t=y}$

$$\begin{cases} x=-2t-\frac{9}{5} \\ y=t \\ z=-x-y=t+\frac{9}{5} \end{cases}$$ parametric

symmetric $$\frac{x+\frac{9}{5}}{-2}=\frac{y-0}{1}=\frac{z-\frac{9}{5}}{1}$$

Ex4 ① $\begin{cases} x+2y+3z=10 & \text{normal } \langle 1,2,3\rangle \\ 3x+6y+9z=0 & \text{normal } \langle 3,6,9\rangle = \langle 1,2,3\rangle \end{cases}$

$\Rightarrow x+2y+3z=10, \quad x+2y+3z=0 \Rightarrow$ parallel

② $\begin{cases} x=2t+3 \\ y=-3t-4 \\ z=5t+7 \end{cases} \quad \begin{cases} x=4s-9 \\ y=8s-3 \\ z=-3s+5 \end{cases}$

direction $\langle 2,-3,5\rangle \quad \langle 4,8,-3\rangle$ not parallel

intersecting? $\begin{cases} 2t+3=4s-9 \Rightarrow 2t=4s-12 \Rightarrow t=2s-6 \\ -3t-4=8s-3 \\ 5t+7=-3s+5 \end{cases}$

$-3(2s-6)-4=8s-3 \Rightarrow -6s+18=8s-3 \Rightarrow 14s=21$

$\Rightarrow s=\frac{3}{2}, \quad t=2s-6=-3$

$5(-3)+7=-8 \qquad -3\left(\frac{3}{2}\right)+5=\frac{1}{2} \qquad$ So $-8 \neq \frac{1}{2}$

no intersection $\Rightarrow$ skew (not parallel or intersect)

③ $\begin{cases} x=2t+3 \\ y=-3t-4 \\ z=5t+7 \end{cases} \quad \begin{cases} x=4s+3 \\ y=-6s-4 \\ z=10s+7 \end{cases}$

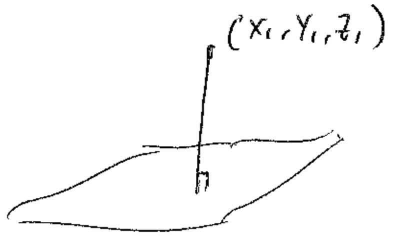
direction $\langle 2,-3,5\rangle \quad \langle 4,-6,10\rangle = 2\langle 2,-3,5\rangle$

$\Rightarrow$ parallel

indeed both contains $(3,-4,7) \Rightarrow$ so same line!

(identical)

Distance from a point $P(x_1, y_1, z_1)$ to $ax+by+cz+d=0$ page 15



a line passing through (x_1, y_1, z_1) and normal to the plane is

$$L: \ x = at + x_1, \quad y = bt + y_1, \quad z = ct + z_1$$

intersection of L and plane is $(x_2, y_2, z_2) = (at+x_1, bt+y_1, ct+z_1)$

$$a(at+x_1) + b(bt+y_1) + c(ct+z_1) + d = 0$$

$$\Rightarrow (a^2+b^2+c^2)\, t = -(ax_1+by_1+cz_1+d)$$

$$\Rightarrow t_1 = -\frac{(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}$$

Distance from (x_1, y_1, z_1) to (x_2, y_2, z_2)

is $D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} = \sqrt{(at_1)^2 + (bt_1)^2 + (ct_1)^2}$

$$= \sqrt{a^2+b^2+c^2} \cdot |t_1| = \sqrt{a^2+b^2+c^2}\ \frac{|ax_1+by_1+cz_1+d|}{a^2+b^2+c^2} = \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

Ex5 ① $P(1,2,3)$ to $5x-7y+z=22$

$$5x-7y+z-22=0$$

$$d = \frac{|5\cdot 1 - 7\cdot 2 + 3 - 22|}{\sqrt{5^2+7^2+1^2}} = \frac{|8-36|}{\sqrt{75}} = \frac{28}{5\sqrt{3}} = \frac{28\sqrt{3}}{15}$$

(2) distance between $5x-7y+z=22$ and $5x-7y+z=1$

take a point on $5x-7y+z=1 \Rightarrow (0,0,1)$

$$d=\frac{|5\cdot 0-7\cdot 0+1-22|}{\sqrt{5^2+7^2+1^2}}=\frac{21}{\sqrt{75}}=\frac{21}{5\sqrt{3}}=\frac{7\sqrt{3}}{5}$$

(3) Find distance between $L_1: \begin{cases} x=2t+3 \\ y=-3t+4 \\ z=5t+7 \end{cases}$ and $L_2: \begin{cases} x=4s-9 \\ y=8s-3 \\ z=-3s+5 \end{cases}$

Strategy: Find two planes P_1 and P_2 parallel to each other

$L_1 \subset P_1$ and $L_2 \subset P_2$

then calculate distance between P_1 and P_2

normal direction of P_1 and P_2 $\langle 2,-3,5\rangle \times \langle 4,8,-3\rangle$

$$= \begin{vmatrix} i & j & k \\ 2 & -3 & 5 \\ 4 & 8 & -3 \end{vmatrix} = -31i+26j+28k$$

P_1: $\langle -31,26,28\rangle \langle x-3, y-4, z-7\rangle = 0$

P_2: $\langle -31,26,28\rangle \langle x+9, y+3, z-5\rangle = 0$