Math 212 Lecture 2

Ex1

(a) $\langle 1,1\rangle \cdot \langle -1,1\rangle = 1\cdot(-1)+1\cdot 1 = 0$

(b) $\langle 2,3\rangle \cdot \langle 2,3\rangle = 4+9 = 13$

(c) $(2i-7j-5k)(-2i+8j+7k) = 2\cdot(-2)+(-7)\cdot 8+(-5\cdot 7)$

$= -4-56-35 = -95$

Ex2

(a) $u = \langle 1,2,3\rangle$, $v = \langle 4,0,-1\rangle$

$$\cos\theta = \frac{u\cdot v}{|u|\cdot|v|} = \frac{4+0-3}{\sqrt{1^2+2^2+3^2}\cdot\sqrt{4^2+1^2}} = \frac{1}{\sqrt{14}\cdot\sqrt{17}}$$

So $\theta = \cos^{-1}\left(\frac{1}{\sqrt{14}\cdot\sqrt{17}}\right)$

(b) $u = \langle 4,6\rangle$, $v = \langle -3,2\rangle$

$u\cdot v = 4\cdot(-3)+6\cdot 2 = 0 \Rightarrow$ orthogonal

(c) $u = i-2j+3k$

$$\cos\alpha = \frac{u\cdot i}{|u|\cdot|i|} = \frac{1}{\sqrt{14}} \qquad \cos\beta = \frac{u\cdot j}{|u|} = \frac{-2}{\sqrt{14}} \qquad \cos\gamma = \frac{3}{\sqrt{14}}$$

$$\alpha = \cos^{-1}\left(\frac{1}{\sqrt{14}}\right) \qquad \beta = \cos^{-1}\left(\frac{-2}{\sqrt{14}}\right) \qquad \gamma = \cos^{-1}\left(\frac{3}{\sqrt{14}}\right)$$

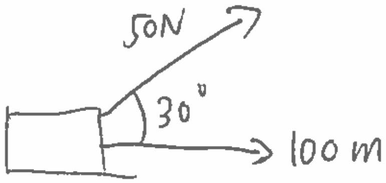
Ex3

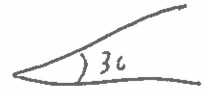
$$\mathrm{Proj}_a b = \frac{a\cdot b}{|a|^2}\,a \qquad V = \langle -1,-2,2\rangle \qquad W = \langle 3,3,4\rangle$$

$$\mathrm{Proj}_W V = \frac{W\cdot V}{|W|^2}\,W = \frac{\langle 3,3,4\rangle\langle -1,-2,2\rangle}{\sqrt{3^2+3^2+4^2}}\langle 3,3,4\rangle$$

$$= \frac{-1}{\sqrt{34}}\langle 3,3,4\rangle = \frac{\langle -3,-3,-4\rangle}{\sqrt{34}} \qquad \mathrm{comp}_W V = -\frac{1}{\sqrt{34}}$$

Ex 4





$$\vec{F} = 50 \langle \cos 30^\circ, \sin 30^\circ \rangle = 50 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$$

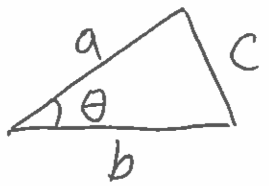
$$\overrightarrow{PQ} = \langle 100, 0 \rangle$$

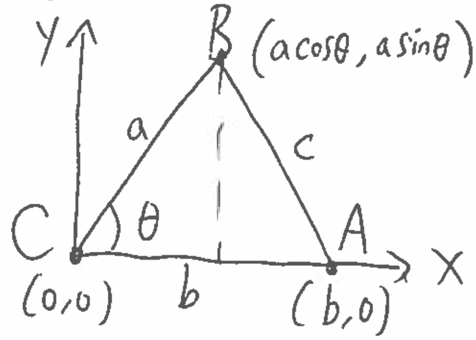
$$W = 50 \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle \cdot \langle 100, 0 \rangle = 50 \cdot \frac{\sqrt{3}}{2} \cdot 100 = 2500\sqrt{3}$$

$(N \cdot m)$

Proof of Law of Cosine   For a triangle with sides a, b, c,

$$c^2 = a^2 + b^2 - 2ab\cos\theta, \quad \text{where } \theta \text{ is the angle opposite to } c$$

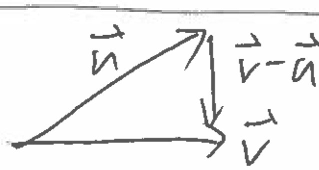




$$(\text{Length of } \overrightarrow{AB}) = c = \sqrt{(a\cos\theta - b)^2 + (a\sin\theta - 0)^2} = \sqrt{a^2\cos^2\theta - 2ab\cos\theta + b^2 + a^2\sin^2\theta}$$

$$= \sqrt{a^2 + b^2 - 2ab\cos\theta} \Rightarrow c^2 = a^2 + b^2 - 2ab\cos\theta$$

Proof of $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|}$



proof $\vec{u}$ and $\vec{v}$ generate a triangle

$$c = |\vec{v} - \vec{u}| \qquad a = |\vec{u}|, \quad b = |\vec{v}|$$

$$\Rightarrow \quad |\vec{v} - \vec{u}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}|\cos\theta$$

$$|\vec{v} - \vec{u}|^2 = (\vec{v} - \vec{u})\cdot(\vec{v} - \vec{u}) = \vec{v}\cdot\vec{v} - \vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{u} + \vec{u}\cdot\vec{u}$$

$$= |\vec{v}|^2 - 2\vec{u}\cdot\vec{v} + |\vec{u}|^2$$

$$\Rightarrow \quad -2\vec{u}\cdot\vec{v} = -2|\vec{u}|\cdot|\vec{v}|\cos\theta$$

$$\Rightarrow \quad \cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|\cdot|\vec{v}|}$$