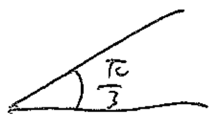
Math 212 Lecture 10

① $D_u f(1,2) = \langle f_x, f_y \rangle \cdot \vec{u} = \langle 5y^2 - 12x^2y,\ 10xy - 4x^3 \rangle$

$\cdot \langle \frac{5}{13}, \frac{12}{13} \rangle = \langle 5\cdot 2^2 - 12\cdot 1^2 \cdot 2,\ 10\cdot 1\cdot 2 - 4\cdot 1^3 \rangle \cdot \langle \frac{5}{13}, \frac{12}{13} \rangle$

$= \langle -4, 16 \rangle \cdot \langle \frac{5}{13}, \frac{12}{13} \rangle = -\frac{20}{13} + \frac{192}{13} = \frac{172}{13}$

② $\theta = \pi/3$



direction $\vec{u} = \langle \cos\frac{\pi}{3}, \sin\frac{\pi}{3} \rangle$
$= \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

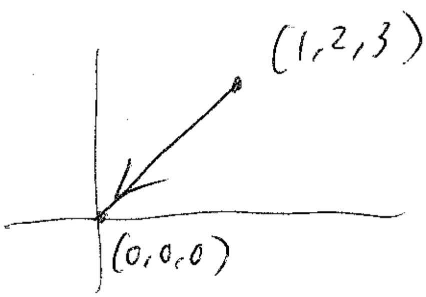
$D_u f(1,2) = \langle f_x, f_y \rangle \cdot \vec{u} = \langle \sin(xy) + x\cos(xy)\cdot y,\ x\cos(xy)\cdot x \rangle$

$= \langle \sin(xy) + xy\cos(xy),\ x^2\cos(xy) \rangle \cdot \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

$= \langle \sin(0) + 0\cdot\cos(0),\ 4\cos 0 \rangle \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

$= \langle 0, 4 \rangle \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle = 2$

③



$\vec{u} = \langle 0-1, 0-2, 0-3 \rangle = \langle -1, -2, -3 \rangle$

unit vector $\vec{u}\ \frac{\langle -1, -2, -3 \rangle}{\sqrt{1^2+2^2+3^2}} = \frac{-1}{\sqrt{14}} \langle 1, 2, 3 \rangle$

$D_u f(1,2,3) = \langle f_x, f_y, f_z \rangle \cdot \vec{u} = \langle 2x, 2y, 2z \rangle \cdot \frac{-1}{\sqrt{14}} \langle 1, 2, 3 \rangle$

$= \langle 2, 4, 6 \rangle \cdot \frac{-1}{\sqrt{14}} \langle 1, 2, 3 \rangle = -\frac{2+8+18}{\sqrt{14}}$

$= \frac{-28}{\sqrt{14}} = -2\sqrt{14}$

① $f(x,y) = x^3 y$

$\nabla f = \langle f_x, f_y \rangle = \langle 3x^2 y, x^3 \rangle \Big|_{(2,3)} = \langle 3\cdot 2^2 \cdot 3, 2^3 \rangle$

$= \langle 36, 8 \rangle$

increasing fastest direction $\dfrac{\nabla f}{|\nabla f|} = \dfrac{\langle 36, 8\rangle}{\sqrt{36^2+8^2}} = \dfrac{\langle 36, 8\rangle}{4\sqrt{85}} = \dfrac{\langle 9, 2\rangle}{\sqrt{85}}$

rate $= \langle 36, 8 \rangle \cdot \dfrac{\nabla f}{|\nabla f|} = \dfrac{36^2+8^2}{\sqrt{36^2+8^2}} = \sqrt{36^2+8^2} = 4\sqrt{9^2+2^2} = 4\sqrt{85}$

So direction $= \dfrac{\langle 9, 2\rangle}{\sqrt{85}}$, rate $= 4\sqrt{85}$.

② $x^2 - 2y^2 + z^2 + yz = 2$ at $(1, 0, 1)$

$f(x,y,z) = x^2 - 2y^2 + z^2 + yz - 2$

tangent plane. $\langle f_x, f_y, f_z \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$.

$\underbrace{\langle 2x_0, -4y_0 + z_0, 2z_0 + y_0 \rangle}_{\text{Substitute}} \langle x-1, y-0, z-1 \rangle = 0$

$\langle 2, 1, 2 \rangle \langle x-1, y-0, z-1 \rangle = 0$

$2(x-1) + 1\cdot(y-0) + 2(z-1) = 0$ $\quad 2x - 2 + y + 2z - 2 = 0$

$2x + y + 2z = 4$

Normal line :

$\langle x, y, z \rangle = \langle x_0, y_0, z_0 \rangle + t \langle f_x, f_y, f_z \rangle$

$= \langle 1, 0, 1 \rangle + t \langle 2, 1, 2 \rangle$

$x = 1 + 2t, \; y = t, \; z = 1 + 2t$