# Math 212 Lecture 1

**Ex 1** (a) $x=0$ $\quad \{(x,y,z)\in\mathbb{R}^3 : x=0,\ y,z\in\mathbb{R}\}$

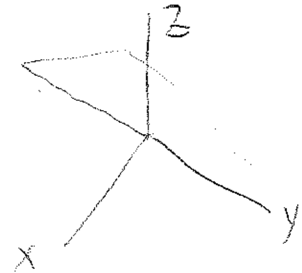
yz-plane

(b) $y=-2$ $\quad \{(x,y,z)\in\mathbb{R}^3 : y=2,\ x,z\in\mathbb{R}\}$

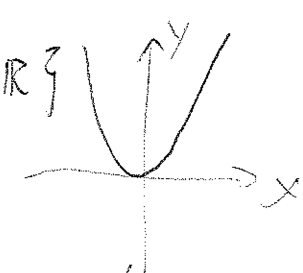
a plane parallel to xz-plane with $y=-2$.

(c) $x-z=0$ $\quad \{(x,y,z)\in\mathbb{R}^3 : x-z=0,\ y\in\mathbb{R}\}$

a plane




(d) $y=x^2$ $\quad \{(x,y,z)\in\mathbb{R}^3 : y=x^2,\ z\in\mathbb{R}\}$




a surface generated by $y=x^2$

(e) $x^2+y^2=4$ $\quad \{(x,y,z)\in\mathbb{R}^3 : x^2+y^2=4,\ z\in\mathbb{R}\}$

a cylinder with axis being z-axis.

**Ex 2** (a) $y\geq 2$ (half space) (b) $1\leq x\leq 4$ (region between two parallel planes)

(c) $x=1$ and $y=4$ (a line) (d) $x^2+y^2<4$ (interior of a cylinder)

(e) $x^2+y^2=4$ and $z=2$ (a circle) (f) $x^2+y^2+z^2=4$ (a sphere)

(g) $x^2+y^2+z^2<4$ (interior of a sphere, or a ball) (h) $x^2+y^2+z^2>4$ (exterior of a sphere)

**Ex3** $P = (-2, 3, 5)$, $Q = (1, 8, -4)$

(a) $|\overline{PQ}| = \sqrt{(1-(-2))^2 + (8-3)^2 + (-4-5)^2} = \sqrt{3^2 + 5^2 + 9^2} = \sqrt{115}$

(b) distance of $P$ to xy-plane : xy-plane ($z = 0$)

closest point on xy-plane to $P$: $(-2, 3, 0)$

So distance $= 5$

(c) distance of $P$ to x-axis; x-axis ($y=0$ and $z=0$)

closest point on x-axis to $P$: $(-2, 0, 0)$

So distance $= \sqrt{3^2 + 5^2} = \sqrt{34}$

**Ex4** (a) $(x-1)^2 + (y-2)^2 + (z-3)^2 = 4$

yz-plane: $x = 0$

intersection: $\{(x, y, z) \in \mathbb{R}^3 : (x-1)^2 + (y-2)^2 + (z-3)^2 = 4 \text{ and } x = 0\}$

$= \{(0, y, z) \in \mathbb{R}^3 : (y-2)^2 + (z-3)^2 = 3\}$

a circle

(b) $\{(x, y, z) : 1 \leq x^2 + y^2 + z^2 \leq 9\}$

the region between two concentric spheres with radius 1 and 3, centered at $(0, 0, 0)$, annulus

(c) $x^2+y^2+z^2 = 4x-2y+10$

$$x^2+y^2+z^2-4x+2y-10=0$$

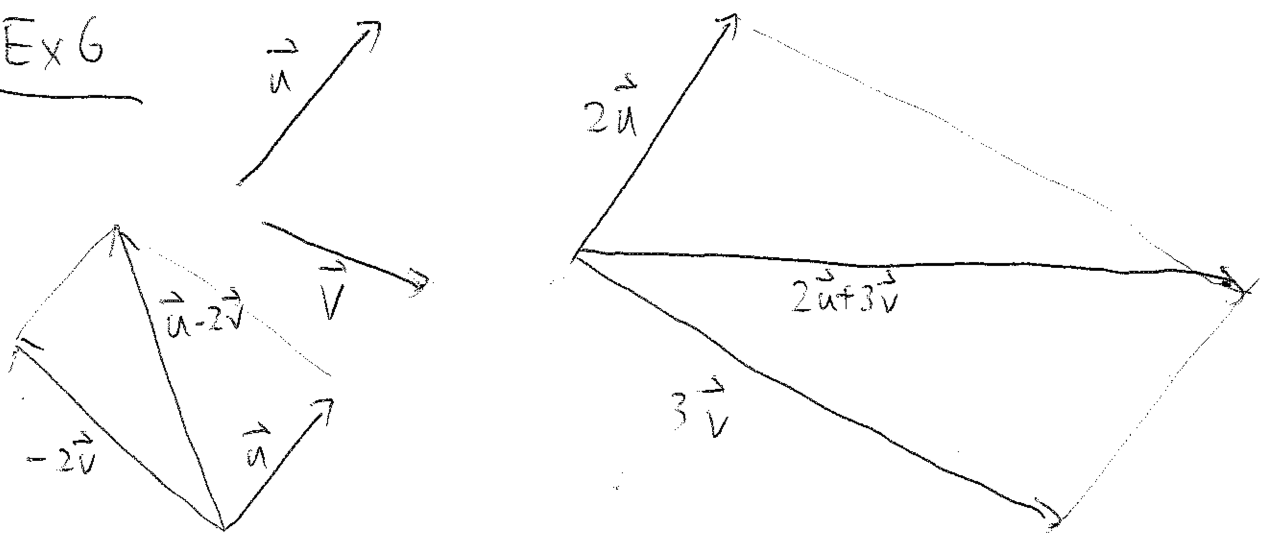
$$(x^2+4x)+(y^2+2y)+z^2-10=0$$

$$(x^2+4x+4)-4+(y^2+2y+1)-1+z^2-10=0$$

$$(x+2)^2+(y+1)^2+(z+0)^2=15$$

center $(-2,-1,0)$, radius $\sqrt{15}$

Ex 6




Ex 7 $\vec{u}=3i-2k$, $\vec{v}=i-j+k$,

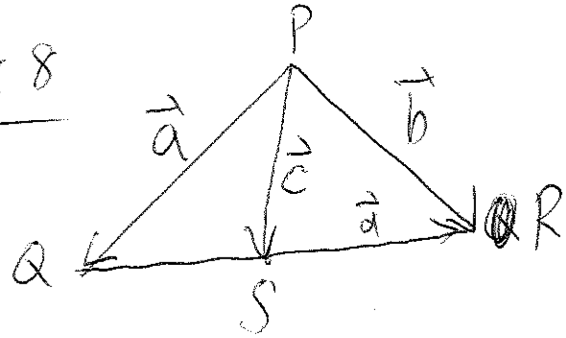
$$3\vec{u}-4\vec{v}=3(3i-2k)-4(i-j+k)=5i+4j-10k$$

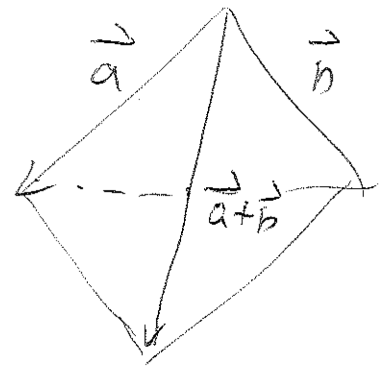
$$|3\vec{u}-4\vec{v}|=\sqrt{5^2+4^2+10^2}=\sqrt{141}$$

$\vec{w}$ = unit vector opposite as $3\vec{u}-4\vec{v}$

$$=\frac{-5i-4j+10k}{\sqrt{141}}$$

Ex 8



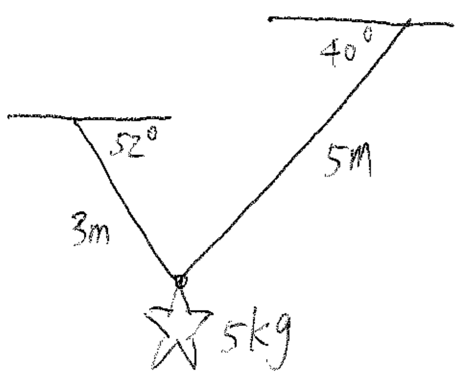

$\vec{c} = \frac{1}{2}(\vec{a}+\vec{b})$

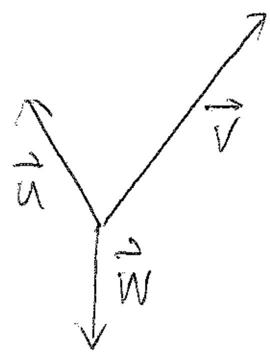
parallelogram law

Since $\vec{c}+\vec{d}=\vec{b}$ then $\vec{d} = \vec{b}-\vec{c} = \vec{b} - (\frac{1}{2}\vec{a} + \frac{1}{2}\vec{b})$

$= \frac{1}{2}\vec{b} - \frac{1}{2}\vec{a}$

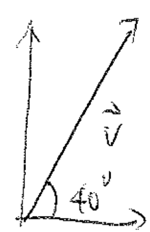
Ex 9




$\vec{w} = (0, -5)$


$\vec{u} = \langle -|\vec{u}|\cos 52^\circ, |\vec{u}|\sin 52^\circ \rangle$




$\vec{v} = \langle |\vec{v}|\cos 40^\circ, |\vec{v}|\sin 40^\circ \rangle$

$$\vec{u}+\vec{v}+\vec{w} = \vec{0}$$

$$\Rightarrow \langle -|\vec{u}|\cos 52^\circ + |\vec{v}|\cos 40^\circ,\ |\vec{u}|\sin 52^\circ + |\vec{v}|\sin 40^\circ - 5 \rangle = \vec{0}$$

$$\Rightarrow |\vec{v}| = \frac{\cos 52^\circ}{\cos 40^\circ}|\vec{u}| \quad \Rightarrow |\vec{u}|\sin 52^\circ + \frac{\cos 52^\circ}{\cos 40^\circ}\sin 40^\circ |\vec{u}| = 5$$

$$\Rightarrow |\vec{u}| = \frac{5\cos 40^\circ}{\sin 52^\circ \cos 40^\circ + \cos 52^\circ \sin 40^\circ} = \frac{5\cos 40^\circ}{\sin 92^\circ}$$

$$|\vec{v}| = \frac{\cos 52^\circ}{\cos 40^\circ} \cdot 5\frac{\cos 40^\circ}{\sin 92^\circ} = 5\frac{\cos 52^\circ}{\sin 92^\circ}$$

tension of 3m wire

$$\vec{u} = \left\langle -\frac{5\cos 40^\circ \cos 52^\circ}{\sin 92^\circ}, \frac{5\cos 40^\circ \sin 52^\circ}{\sin 92^\circ} \right\rangle$$

tension of 5m wire

$$\vec{v} = \left\langle \frac{5\cos 52^\circ \cos 40^\circ}{\sin 92^\circ}, \frac{5\sin 40^\circ \cos 52^\circ}{\sin 92^\circ} \right\rangle$$

Ex 10 $\vec{b} = \langle 1, 1 \rangle$ , $\vec{c} = \langle -1, 2 \rangle$

$$\vec{a} = \langle a_1, a_2 \rangle = s\langle 1, 1 \rangle + t\langle -1, 2 \rangle$$

$$\Rightarrow \begin{aligned} a_1 &= s - t \\ a_2 &= s + 2t \end{aligned} \quad \Rightarrow a_2 - a_1 = 3t \Rightarrow t = \frac{a_2 - a_1}{3}$$

$$s = a_1 + t = a_1 + \frac{a_2 - a_1}{3} = \frac{a_2 + 3a_1}{3}$$

So for any vector $\vec{a} = \langle a_1, a_2 \rangle$

take $s = \dfrac{a_2 + 3a_1}{3}$ and $t = \dfrac{a_2 - a_1}{3}$

we have $\vec{a} = s\vec{b} + s\vec{c}$