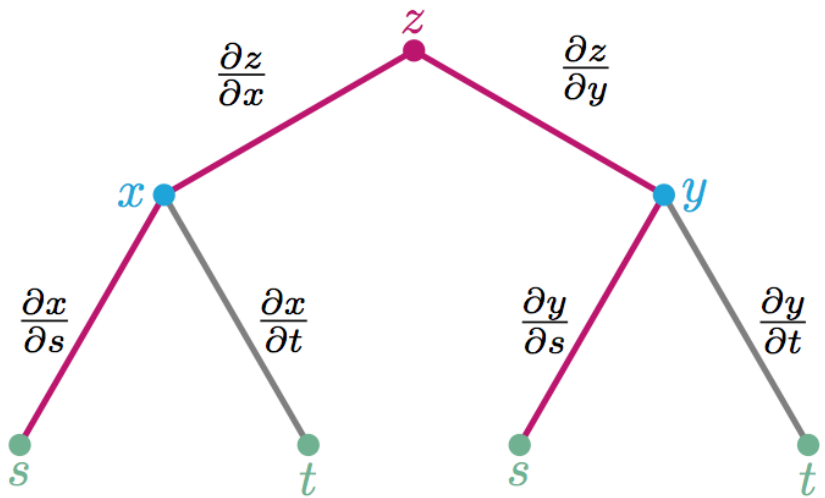
Chain Rule

Chain Rule in 1-d:
If $y = f(g(x))$, then $y' = f'(g(x)) \cdot g'(x)$;
or $y = f(z)$, $z = g(x)$, then $\dfrac{dy}{dx} = \dfrac{df}{dz} \cdot \dfrac{dg}{dx}$

Chain Rule in 2-d:
$z = f(x, y)$, $x = g(t)$, $y = h(t)$
$$\frac{dz}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$$

$z = f(x, y)$, $x = g(s, t)$, $y = h(s, t)$
$$\frac{dz}{ds} = \frac{df}{dx} \cdot \frac{dx}{ds} + \frac{df}{dy} \cdot \frac{dy}{ds}, \quad \frac{dz}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$$



Examples

Examples:
(1) Find z_t if $z = \sqrt{x^2 + y^2}$, $x = e^{2t}$, $y = e^{2t}$.
(2) Find z_s and z_t if $z = e^{xy} \tan y$, $x = s + 2t$ and $y = s/t$.
(3) Find w_t and w_u if $w = f(x, y, z)$, $x = x(t, u)$, $y = y(t, u)$, $z = z(t, u)$.
(4) Find z_x and z_y if $xyz = \cos(x + y + z)$
(5) Car A is traveling north on Highway 16 and Car B is traveling west on Highway 83. each car is approaching the intersection of these highways. At a certain moment, car A is 0.3 km from the intersection, and traveling at 90 km/h, while car B is 0.4 km from the intersection and traveling at 80 km/h. how fast is the distance between the cars changing at the moment?