Equation of a line in 2-d

1. \( y = k(x - x_0) + y_0 \) (coordinate equation: point-slope) \( k \): slope,

2. \( x = At + x_0, \ y = Bt + y_0 \), (parametric equation)
   \( \langle A, B \rangle \): direction of line

3. \( \langle a, b \rangle \cdot \langle x - x_0, y - y_0 \rangle = 0 \),
   (vector equation) \( \langle a, b \rangle \): normal direction
   (meaning: all points such that \( \langle x - x_0, y - y_0 \rangle \) is orthogonal to \( \langle a, b \rangle \))

4. \( ax + by = c \) (a general form)

Example 1: Find the three forms of the equations of the line passing thorough \((1, 2)\) and \((-3, 7)\).
Equation of a plane in 3-d

vector equation: \( \langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0 \)
\( \langle a, b, c \rangle \): normal vector, \((x_0, y_0, z_0)\) a point on the plane
(one point and one direction can determine a plane)

a general form: \( ax + by + cz = d \) (\( \langle a, b, c \rangle \): normal vector)

Example 2:

1. Find the equation of the plane that passes through the points \( P(1, 2, 3) \), \( Q(2, -4, 9) \) and \( R(0, 5, -4) \).
2. Find the angle between the planes \( x - y + z = 4 \) and \( 3x + 4y - 5z = 0 \).
3. Find the equation of the plane through the point \( P(1, 2, 3) \) and parallel to \( 2x + 4y + 7z = 18 \)
Equation of a line in 3-d

Parametric equation: \( x = At + x_0, \ y = Bt + y_0, \ z = Ct + z_0 \)
\( \langle A, B, C \rangle \): direction of line

Vector version: \( \mathbf{r} = \mathbf{r}_0 + t \mathbf{v} \), where \( \mathbf{r}_0 \) is a vector from origin to one of the points on the line, and \( \mathbf{v} \) is the direction of the line.

Symmetric equation: \( t = \frac{x - x_0}{A} = \frac{y - y_0}{B} = \frac{z - z_0}{C} \)
(\( A \neq 0, \ B \neq 0, \ C \neq 0 \))

Example 3:
1. Find the parametric and symmetric equations of the line passing through \( P(1, 2, 3) \) and \( Q(-4, 7, 0) \).
2. Find the parametric and symmetric equations of the line of z-axis.
3. Find the equation of the line where the planes \( x + y + z = 0 \) and \( 2x - 3y + 7z = 9 \) intersect.

Summary:
Equation of plane: \( \overrightarrow{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0 \), where \( \overrightarrow{n} \) is the normal vector, \( \mathbf{r}_0 \) is a point on the plane.
Equation of the line: \( \mathbf{r} - \mathbf{r}_0 = t \overrightarrow{n} \), where \( \mathbf{r}_0 \) is a point on the line and \( \overrightarrow{n} \) is the direction of the line.
The relations of lines and planes

In 2-d, two lines are either parallel or intersecting
In 3-d, two planes are either parallel or intersecting
(parallel if their normal vectors are parallel)
In 3-d, two lines can be parallel, intersecting, or neither (skew)

Example 4:

1. Determine the relation of planes \( x + 2y + 3z = 10 \) and \( 3x + 6y + 9z = 0 \)

2. Determine the relation of lines \( x = 2t + 3, \ y = −3t − 4, \ z = 5t + 7 \) and 
   \( x = 4s − 9, \ y = 8s − 3, \ z = −3s + 5 \).

3. Determine the relation of lines \( x = 2t + 3, \ y = −3t − 4, \ z = 5t + 7 \) and 
   \( x = 4s + 3, \ y = −6s − 4, \ z = 10s + 7 \).
Distances

The distance from a point $P(x_1, y_1, z_1)$ to a plane $ax + by + cz + d = 0$:

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

**Example 5:**

1. Find the distance from $P(1, 2, 3)$ to the plane $5x - 7y + z = 22$.
2. Find the distance between two parallel planes $5x - 7y + z = 22$ and $5x - 7y + z = 1$.
3. Find the distance between the lines $x = 2t + 3, y = -3t - 4, z = 5t + 7$ and $x = 4s - 9, y = 8s - 3, z = -3s + 5$. 