Final Exam

Final Exam: May 1 (Tuesday), 9am-12noon, Small 233
any time change must be approved by Dean of Students
2 sample exam and solution keys are at Blackboard website

Final Exam Office Hours:
April 29 (Sunday) 2-3:30pm, Jones Hall 117
April 30 (Monday) 2-5pm, Jones Hall 117

Composition of the exam:

Chapter 12-13 $\approx$ 30%, Chapter 14 $\approx$ 30%, Chapter 15 $\approx$ 40%

About $15 - 16$ problems.
format similar to sample final exam

Some formulas will be available to you (trigonometric formulas, integrals), see sample final exam

Chapters 12-13

Sections 12.1-12.5, Section 13.1-13.4 (not including curvature)

- 2-D and 3-D coordinate systems, distance formula, vector
- dot product, cross product, angle between vectors, projection of a vector along another vector,
- Area of parallelogram spanned by two vectors, volume of parallelepiped spanned by three vectors (triple product),
- Equation of circle, sphere, line (parametric, symmetric, vector), plane (vector, linear equation), parallel or skew lines, parallel planes, distance from a point to plane
- direction of a line, normal direction of a plane
- cylinder
- vector functions (space curve), limit, derivative and integral of vector functions, tangent line of space curve, arc length,
- velocity, acceleration, speed, projectile problem,

Chapter 14

Sections 14.1-14.8

- Multi-variable functions, domain, level curves, level surfaces,
- limits and continuity of multi-variable functions,
- Partial derivatives (1st, 2nd orders), implicit differentiation, chain rule,
- tangent plane equation, linear approximation, differential, estimate errors using differential,
- directional derivatives, gradient, maximum directional derivative,
- (optimization) local maximum (minimum) point, saddle point, 1st derivative test, 2nd derivative test, absolute minimum (maximum) values, shortest distance problem,
- (constraint optimization) Lagrange multiplier method,

Chapter 15

Sections 15.1-15.4, 15.7-15.10 (no 15.5 application, 15.6 surface area)

- Double integral on a rectangle or on a general domain, area of a region
- Iterated integral on a rectangle or on a general domain (type I and II),
- Evaluate double integral by using iterated integral (expressing the region is the key)
- polar coordinate, integral with polar coordinate
- Triple integral on a rectangular box or on a general domain, volume of a region
- Iterated integral on a rectangular box or on a general domain (type 1, 2 and 3),
- Cylindrical coordinate system, spherical coordinate system
- Change of variables, Jacobian (2D and 3D), integrals under change of variables,

Geometry

2D

- circle: $(x-a)^2+(y-b)^2=r^2$, upper half: $y-b=\sqrt{r^2-(x-a)^2}$
- line: $y=kx+b$ or $ax+by+c=0$ or $(x,y)=(a,b)t+(x_0,y_0)$
- parabola: $y=a(x-b)^2+c$ or $x=a(y-b)^2+c$
- parallelogram: $\{(x,y): A\le ax+by\le B, C\le cx+dy\le D\}$
- triangle: connecting three points, half of the parallelogram

3D

- sphere: $(x-a)^2+(y-b)^2+(z-c)^2=r^2$,
- line: $(x,y,z)=(a,b,c)t+(x_0,y_0,z_0)$
- plane: $ax+by+cz+d=0$
- circular cylinder: $(x-a)^2+(y-b)^2=r^2$ in $\mathbb{R}^3$
- circular cone: $z^2=a^2(x^2+y^2)$ or $z=a\sqrt{x^2+y^2}$
- parallelepiped: $\{(x,y,z): A\le ax+by+dz\le B, C\le dx+ey+fz\le D, E\le gx+hy+iz\le F\}$
- tetrahedra: connecting four points, 1/6 of the parallelepiped

Polar, Cylindrical and spherical coordinates

Polar: $x = r\cos\theta$, $y = r\sin\theta$, $r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$ (or $\tan\theta = y/x$)

Cylindrical: $x = r\cos\theta$, $y = r\cos\theta$, $z = z$ (no change in z!)
$r^2 = x^2 + y^2$, $\tan\theta = \dfrac{y}{x}$, $z = z$.

$$\iiint_E f(x,y,z)dV = \iint_D \left[\int_{u_1(x,y)}^{u_2(x,y)} f(x,y,z)dz\right] dA$$
$$= \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z) r dz dr d\theta$$

Spherical: $x = \rho\sin(\phi)\cos(\theta)$, $y = \rho\sin(\phi)\sin(\theta)$, $z = \rho\cos(\phi)$
$r = \sqrt{x^2 + y^2} = \rho\sin(\phi)$, $z = \rho\cos(\phi)$
$\rho^2 = x^2 + y^2 + z^2$, $\tan(\phi) = \dfrac{r}{z}$, $\tan\theta = \dfrac{y}{x}$,
(θ is the angle in xy plane, ϕ is the angle between positive z-axis and vector $\langle x, y, z\rangle$).

$$\iiint_E f(x,y,z)dV =$$
$$= \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(\theta,\phi)}^{u_2(\theta,\phi)} f(\rho\sin(\phi)\cos(\theta), \rho\sin(\phi)\sin(\theta), \rho\cos(\phi))\rho^2 \sin\phi \, d\rho d\phi d\theta$$