Math 442 Homework 9: (due April 9, Monday, 2018)

1. Page 209 Problem 1:

   (a) Solve the problem $u_t = u_{xx}$ in the interval $[0, 4]$ with $u = 0$ at both ends and $u(x, 0) = x(4 - x)$, using the forward difference scheme with $\Delta x = 1$ and $\Delta t = 0.25$. Calculate four time steps (up to $t = 1$).

   (b) Do the same with $\Delta x = 0.50$ and $\Delta t = 0.0625 = 1/16$. Calculate four time steps (up to $t = 0.25$).

   (c) Compare your answers with each other. How close are they at $x = 2.0$ and $t = 0.25$?

2. Page 209 Problem 2: Do the same with $\Delta x = 1$ and $\Delta t = 1$. Calculate by hand or by computer up to $t = 7$.

3. Page 209 Problem 4: Solve by hand the problem $u_t = u_{xx}$ in the interval $[0, 1]$ with $u_x = 0$ at both ends. Use the forward scheme (2) for the PDE, and the scheme (14) for the boundary conditions. Assume $\Delta x = 1/5$, $\Delta t = 1/100$, 100, and start with the initial data: 0 0 64 0 0 0. Compute for four time steps.

4. Page 209 Problem 8:

   (a) Write down the Crank-Nicolson scheme ($\theta = 1/2$) for $u_t = u_{xx}$.

   (b) Let $\Delta x = \Delta t = 1/6$. Let the initial data be 0 0 0 1 0 0 0. Compute the solution by the Crank-Nicolson scheme for one time step ($t = 1/6$). (zero Dirichlet boundary condition is assumed)

5. Page 209 Problem 13: Consider the following scheme for the diffusion equation:

$$\frac{u_j^{n+1} - u_j^{n-1}}{2\Delta t} = \frac{u_{j+1}^n + u_{j-1}^n - u_j^{n+1} - u_j^{n-1}}{(\Delta x)^2}.$$

   It uses a centered difference for u_t and a modified form of the centered difference for u_{xx}.

   (a) Solve it for u_j^{n+1} in terms of s and the previous time steps.

   (b) Show that it is stable for all s. (optional)