Math 442 Homework 8: (due April 2, Monday, 2018)

1. Page 164 Problem 1: Solve $u_{xx} + u_{yy} = 0$ in the rectangle $0 < x < a$, $0 < y < b$ with the following boundary conditions:

$$\begin{aligned} u_x = -a, \quad \text{on} \quad x = 0,\ u_x = 0, \quad \text{on} \quad x = a, \\ u_y = b, \quad \text{on} \quad y = 0,\ u_y = 0, \quad \text{on} \quad y = b. \end{aligned}$$

   (Hint: A shortcut is to guess that the solution might be a quadratic polynomial in x and y.)

2. Page 164 Problem 3: Find the harmonic function $u(x, y)$ in the square $\{0 < x < \pi,\ 0 < y < \pi\}$ with the boundary conditions $u_y(x, 0) = 0$, $u_y(x, \pi) = 0$, $u(0, y) = 0$, $u(\pi, y) = \cos^2 y = (1 + \cos(2y))/2$.

3. Page 172 Problem 2: Solve $u_{xx} + u_{yy} = 0$ in the disk $\{r < a\}$ with the boundary condition $u = 1 + 3\sin\theta$ on $r = a$.

4. Page 176 Problem 5a: Find the steady-state temperature distribution inside an annular plate $\{1 < r < 2\}$, whose outer edge ($r = 2$) is insulated ($\partial u/\partial r = 0$), and on whose inner edge ($r = 1$) the temperature is maintained as $\sin^2\theta$. (Find explicitly all the coefficients, etc.)

5. Page 176 Problem 6: Find the harmonic function $u(r, \theta)$ in the semidisk $\{r < 1,\ 0 < \theta < \pi\}$ with u vanishing on the diameter ($\theta = 0, \pi$) and $u = \pi\sin\theta - \sin 2\theta$ on $r = 1$.