

Math 442 Homework 7: (due March 23, 2018)

1. Page 134 Problem 1: $\sum_{n=0}^{\infty} (-1)^n x^{2n}$ is a geometric series.

- (a) Does it converge pointwise in the interval $-1 < x < 1$?
- (b) Does it converge uniformly in the interval $-1 < x < 1$?
- (c) Does it converge in the L^2 sense in the interval $-1 < x < 1$?

(Hint: You can compute the partial sum explicitly.)

2. Page 134 Problem 7: Let

$$\phi(x) = \begin{cases} -1 - x, & -1 < x < 0; \\ +1 - x, & 0 < x < 1. \end{cases}$$

- (a) Find the full Fourier series of $\phi(x)$ in the interval $[-1, 1]$.
 - (b) Find the first three nonzero terms explicitly.
 - (c) Does it converge in the mean square sense?
 - (d) Does it converge pointwise?
 - (e) Does it converge uniformly to $\phi(x)$ in the interval $-1 < x < 1$?
3. Page 160 Problem 2: Find the solutions that depend only on r of the equation $u_{xx} + u_{yy} + u_{zz} = k^2 u$, where k is a positive constant. (Hint: Substitute $u = v/r$)
4. Page 160 Problem 4: Solve $u_{xx} + u_{yy} + u_{zz} = 0$ in the spherical shell $0 < a < r < b$ with the boundary conditions $u = A$ on $r = a$, and $u = B$ on $r = b$, where A and B are constants. (Hint: look for a solution depending only on r)
5. Page 160 Problem 6: Solve $u_{xx} + u_{yy} = 1$ in the annulus $a < r < b$ with $u(x, y)$ vanishing on both parts of the boundary $r = a$ and $r = b$.