

**Math 442 Homework 6:** (due March 16, 2018)

1. Page 111 Problem 2: Let  $\phi(x) \equiv x^2$  for  $0 \leq x \leq 1 = l$ .
  - (a) Calculate its Fourier sine series.
  - (b) Calculate its Fourier cosine series.

2. Page 111 Problem 4: Find the Fourier cosine series of the function  $|\sin x|$  in the interval  $(-\pi, \pi)$ . Use it to find the sums:  $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$  and  $\sum_{n=1}^{\infty} \frac{(-1)^n}{4n^2 - 1}$ .

3. Page 112 Problem 8: A rod has length  $l = 1$  and constant  $k = 1$ . Its temperature satisfies the heat equation. Its left end is held at temperature 0, and its right end at temperature 1. Initially (at  $t = 0$ ) the temperature is given by

$$\phi(x) = \begin{cases} 5x/2, & \text{for } 0 < x < 2/3, \\ 3 - 2x, & \text{for } 2/3 < x < 1. \end{cases}$$

Find the solution, including the Fourier coefficients. (Hint: first find the equilibrium solution  $U(x)$ , and then solve the heat equation with initial condition  $u(x, 0) = \phi(x) - U(x)$ .)

4. Page 118 Problem 11: Find the Fourier full series of  $e^x$  on  $(-l, l)$  in its real and complex form. (Hint: it is convenient to find the complex form first.)
5. Page 118 Problem 12: Find the Fourier full series of  $\cosh(x)$  on  $(-l, l)$  in its real and complex form. (Hint: Use the result for  $e^x$ . What will be the result for  $e^{-x}$ ?)