Math 442 Homework 3: (due February 9, 2018)

1. Page 38 (1): Solve
$$u_{tt} = c^2 u_{xx},\ u(x,0) = e^x,\ u_t(x,0) = \sin x.$$

2. Page 38 (2): Solve
$$u_{tt} = c^2 u_{xx},\ u(x,0) = \log(1+x^2),\ u_t(x,0) = 4 + x.$$

3. Page 38 (3): The midpoint of a piano string of tension T, density ρ, and length l is hit by a hammer whose head diameter is $2a$. A flea is sitting at a distance $l/4$ from one end. (assume that $a < l/4$; otherwise, poor flea!) How long does it take for the disturbance to reach the flea?

4. Page 38 (8): A spherical wave is a solution of the three-dimensional wave equation of the form $u(r,t)$, where r is the distance to the origin (the spherical coordinate). The wave equation takes the form
$$u_{tt} = c^2 \left(u_{rr} + \frac{2}{r} u_r \right)$$
   (a) Change the variables $v = ru$ to get the equation for v: $v_{tt} = c^2 v_{rr}$.
   (b) Solve for v using (2.1.3) and thereby solve the spherical wave equation.
   (c) Use (2.1.8) to solve it with initial conditions $u(r,0) = \phi(r)$, $u_t(r,0) = \psi(r)$, take both $\phi(r)$ and $\psi(r)$ to be even functions or r.

5. Page 38 (9): Solve
$$u_{xx} - 3u_{xt} - 4u_{tt} = 0,\ u(x,0) = x^2,\ u_t(x,0) = e^x.$$
   (Hint: Factor the operator as we did for the wave equation)

6. Page 41 (2): For a solution $u(x,t)$ of the wave equation with $\rho = T = c = 1$, the energy density is defined as $e = \frac{1}{2}(u_t^2 + u_x^2)$ and the momentum density as $p = u_t u_x$.
   (a) Show that $\partial e/\partial t = \partial p/\partial x$ and $\partial p/\partial t = \partial e/\partial x$.
   (b) Show that both $e(x,t)$ and $p(x,t)$ also satisfy the wave equation.

7. Page 41 (5): For the damped string, equation (1.3.3), show that the energy decreases.