Math 442 Homework 2: (due February 2, 2018)

1. Solve the age-structure population equation with mortality using the coordinate method: (here $u(a,t)$ is the population density at age a and time t)

$$\begin{cases} u_t(a,t) + u_a(a,t) - u(a,t) = 0, & x > 0,\ t > 0, \\ u(0,t) = 100 + 0.1t, & t > 0, \\ u(a,0) = e^{-(a-40)^2}, & a > 0. \end{cases}$$

2. Page 10 (10) Solve $u_x + u_y + u = e^{x+2y}$ with $u(x,0) = 0$. (Hint: use coordinate method)

3. Page 19 (4) Suppose that some particles which are suspended in a liquid medium would be pulled down at the constant velocity $V > 0$ by gravity in the absence of diffusion. Taking account of the diffusion, find the equation for the concentration of particles. Assume homogeneity in the horizontal directions x and y. Let the z axis point upwards.

4. Page 24 (1) By trial and error, find a solution of (i) the diffusion equation $u_t = u_{xx}$; (ii) the wave equation $u_{tt} = u_{xx}$; and (iii) the Laplace equation $u_{xx} + u_{yy} = 0$, with the initial condition $u(x,0) = x^2$. (Hint: look for a solution in the form $u(x,t) = f(x) + g(t)$ or $u(x,y) = f(x) + g(y)$.)

5. Page 25 (3) A Homogeneous body occupying the solid region D is completely insulated. Its initial temperature is $f(\mathbf{x})$. Find the steady state temperature that is reaches after a long time. (Hint: No heat is gained or lost, and let's assume D is one-dimension, so $D = (0, L)$. Insulated means that the boundary condition is no-flux one: $u_x(0,t) = u_x(L,t) = 0$.)

6. Page 25 (4) A rod occupying the interval $0 \le x \le l$ is subject to the source $f(x) = 0$ for $0 < x < l/2$, and $f(x) = H$ for $l/2 < x < l$ where $H > 0$. The rod has physical constant $c = \rho = \kappa = 1$, and its ends are kept at zero temperature.

   (a) Find the steady state temperature of the rod, which is a steady state solution of

$$\begin{aligned} &u_t = u_{xx} + f(x), \quad 0 < x < l, t > 0, \\ &u(0,t) = 0, \ \ u(l,t) = 0, \quad t > 0. \end{aligned}$$

   (b) Which point is the hottest, and what is temperature there?

7. Page 27 (3) Solve the boundary value problem $u'' = 0$ for $0 < x < 1$ with $u'(0) + ku(0) = 0$ and $u'(1) \pm ku(1) = 0$. Do the $+$ and $-$ cases separately. What is special about the case $k = 2$?