

**Math 442 Homework 1:** (due January 26, 2018)

1. Page 5 (2) Which of the following operators are linear?
  - (a)  $\mathcal{L}u = u_x + xu_y$
  - (b)  $\mathcal{L}u = u_x + uu_y$
  - (c)  $\mathcal{L}u = u_x + u_y^2$
  - (d)  $\mathcal{L}u = u_x + u_y + 1$
  
2. Page 5 (3) For each of the following equations, state the order and whether it is nonlinear, linear homogenous, or linear inhomogeneous; provide reasons.
  - (d)  $u_{tt} - u_{xx} + x^2 = 0$
  - (e)  $iu_t - u_{xx} + u/x = 0$
  - (f)  $u_x(1 + u_x^2)^{-1/2} + u_y(1 + u_y^2)^{-1/2} = 0$
  
3. Page 6 (12) Verify by direct substitution that  $u_n(x, y) = \sin(nx) \sinh(ny)$  is a solution of  $u_{xx} + u_{yy} = 0$  for every  $n > 0$ .
  
4. Page 9 (1) Solve the first order equation  $2u_t + 3u_x = 0$  with auxiliary condition  $u(0, x) = \sin x$ .
  
5. Page 10 (3) Solve the equation  $(1 + x^2)u_x + u_y = 0$ . Sketch some of the characteristic curves.
  
6. Page 10 (6) Solve the equation  $\sqrt{1 - x^2}u_x + u_y = 0$  with the condition  $u(0, y) = y$ .
  
7. Page 10 (9) Solve the equation  $u_x + u_y = 1$ . (Hint: Find a particular solution  $u_p(x, y)$ , and find general solution of the homogeneous equation  $u_x + u_y = 0$ .)