Math 442 Homework 1: (due January 29, 2016)

1. Page 5 (2) Which of the following operators are linear?
   (a) $L u = u_x + xu_y$
   (b) $L u = u_x + uu_y$
   (c) $L u = u_x + u_y^2$
   (d) $L u = u_x + u_y + 1$

2. Page 5 (3) For each of the following equations, state the order and whether it is nonlinear, linear homogenous, or linear inhomogeneous; provide reasons.
   (d) $u_{tt} - u_{xx} + x^2 = 0$
   (e) $iu_t - u_{xx} + u/x = 0$
   (f) $u_x(1 + u_x^2)^{-1/2} + u_y(1 + u_y^2)^{-1/2} = 0$

3. Page 6 (12) Verify by direct substitution that $u_n(x, y) = \sin(nx) \sinh(ny)$ is a solution of $u_{xx} + u_{yy} = 0$ for every $n > 0$.

4. Page 9 (1) Solve the first order equation $2u_t + 3u_x = 0$ with auxiliary condition $u(0, x) = \sin x$.

5. Page 10 (3) Solve the equation $(1 + x^2)u_x + u_y = 0$. Sketch some of the characteristic curves.

6. Page 10 (6) Solve the equation $\sqrt{1 - x^2}u_x + u_y = 0$ with the condition $u(0, y) = y$.

7. Page 10 (9) Solve the equation $u_x + u_y = 1$. (Hint: Find a particular solution $u_p(x, y)$, and find general solution of the homogeneous equation $u_x + u_y = 0$.)