(A-1) Let \( r \) be a number such that \( r + 1/r \) is an integer. Prove that for every positive integer \( n \), \( r^n + 1/r^n \) is an integer.

(A-2) We need to put \( n \) cents of stamps on an envelop, but we have only (an unlimited supply of) 5 cents and 12 cents stamps. Prove that we can perform the task if \( n \geq 44 \).

(A-3) Show that for all \( n \), \( 2^{3n} + 1 \) is divisible by \( 3^{n+1} \).

(A-4) \( ** \) Using that \( e = \sum_{n=0}^{\infty} \frac{1}{n!} \) to prove \( e \) is not rational.

(A-5) Which of the following subset of \( \mathbb{R} \) is a field under normal addition and multiplication in \( \mathbb{R} \)?

\[ \begin{align*}
T_1 &= \{ a\sqrt{2} + b\sqrt{3} : a, b \in \mathbb{Q} \}; \\
T_2 &= \{ a + b\sqrt{3} : a, b \in \mathbb{Q} \}; \\
T_3 &= \{ a + b\pi : a, b \in \mathbb{Q} \}.
\end{align*} \]

(A-6) \( ** \) Prove that (O4) and (O5) in the order properties is necessary: show that there is a way to define an “order” for \( \mathbb{Q} \) which satisfies (O1)-(O3), but not (O4) and/or (O5). (You can give same or different examples for (O4) and/or (O5).)

Homework 1: due Jan 29 (Thursday) 5pm,
Required problems: 1.7, 1.11, 2.3, 2.5, 3.1(b), 3.3, 3.4, 3.5, A-1, A-5

General Rule for Homework (apply to all homework assignment, unless other specified):

1. Each homework assignment is 10 points. 5 points for completion, 4 points for correctness, and 1 point as award. Extra points are possible.

2. Each homework assignment has two parts: required problems and optional problems. If you solve all required problems correctly, then you get 9 points. If you also solve \( n \) optional problems correctly, then you get \( 9 + n \) points.

3. You can skip some required problems if you feel they are too easy, but then you have to solve equal number of optional problems.