Project 1

Math 490

This project is to read some material related to reaction-diffusion equations and make a presentation for about 20-25 minutes. The project is to be done by a two-person group. Either or both person in the group can do the presentation, but each student is required to appear at least one presentation for projects 1 and 2. You can use blackboard or/and computer in your presentation.

The following are the introduction, hints and references of each topic.

1. **Eigenvalues and eigenvectors for Robin boundary problem**

   Solve problem Chapter 2 number 3, and explain the physical/biological meaning of the model.

   Reference:

   page 90-100, Partial differential equations, an introduction, by Walter A. Strauss, John Wiley & Sons, Inc, (1992).

2. **Moving fuel patch**

   Solve problems Chapter 3 number 3 and 4.

3. **Diffusion and Probability**

   Consider the probability of the location of a particle in a random walk. One can use the central limit theorem in basic statistics to show that the asymptotic probability distribution is a normal distribution with zero mean (if initially the particle is at zero) and a standard deviation of $\sqrt{2Dt}$, where D is the the limit of $(\Delta x)^2/\Delta t$.

   Reference:

   Your statistics book with introduction of random walk and central limit theorem

   page 19-21, Spatial Ecology via Reaction-Diffusion Equations. By Stephen Cantrell, Christopher Cosner, Wiley, John & Sons, Inc., (2003).

4. **Diffusion from a continuous source with constant flux**

   Derive the solution formula, and use Maple to show the solution graph and simulation of the solution

   References:

   page 341-342, Growth and Diffusion Phenomena: Mathematical Frameworks and Applications. By Robert Banks, Springer-Verlag, New York, (1993).

5. **Nonlinear diffusion and animal dispersal**

   Derive a diffusion equation with nonlinear diffusion from animal dispersal, and find the analytic solution with point release initial condition.

   References:

   page 339-343, Elements of Mathematical Ecology. By Mark Kot, Cambridge University Press, (2001).

   page 402-405, Mathematical Biology, Vol. 1: An Introduction. By James Dickson Murray, Springer-Verlag, New York, (2002).

6. **Random walk with attraction between individuals and aggregation**

   Derive the nonlinear diffusion equation, and explain the biological meaning.

   References:

   page 109-112, Quantitative Analysis of Movement: Measuring and Modeling Population Redistribution in Animals and Plants. By Peter Turchin, Sinauer Associates, Inc, (1998).

   Turchin P (pdf file on class website) Population consequences of aggregative movement. Journal of Animal Ecology 58, (1989), 75-100.

7. **Flows of CO_2 and water internal to a leaf**

   Derive a linear reaction diffusion equation to analyze steady-state CO_2 and water concentration patterns within leaves of different morphologies

   References:

   Parkhurst, David F. (SWEM library) A three-dimensional model for CO_2 uptake by continuously distributed mesophyll in leaves. J. Theoret. Biol. 67 (1977), no. 3, 471–488.

   Gross, Louis J. (pdf file on class website) On the dynamics of internal leaf carbon dioxide uptake. J. Math. Biol. 11 (1981), no. 2, 181–191

8. **Telegraph equation and correlated random walk**

   Derive a partial differential equation by using the correlated random walk, i.e. the walker may be in favor of going to the direction in which it has successful experience.

   References:

   Goldstein, S. (SWEM library offsite) On diffusion by discontinuous movements, and on the telegraph equation. Quart. J. Mech. Appl. Math. 4, (1951). 129–156.

   page 88-92, Quantitative Analysis of Movement: Measuring and Modeling Population Redistribution in Animals and Plants. By Peter Turchin, Sinauer Associates, Inc, (1998).

   Holmes, E.E, (SWEM Library) Are diffusion models too simple? A comparison with telegraph models of invasion. American Naturalist 142, (1993), 779-795.

9. **A diffusion genetic model**

   References:

   Nagylaki, Thomas (pdf file, available soon) A diffusion model for geographically structured populations. J. Math. Biol. 6 (1978), no. 4, 375–382.